Instructional model for understanding roof ponding

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Volume 51 (2006), issue 2/3, of HERON was a Special Issue 'Ponding of Roof Structures' with contributions from the universities in Delft and Eindhoven and parties from the business community. It appeared that a major difference occurs between very flexible flat roof systems and stiffer ones, and also behaviour differs between flat roofs with and without slopes. At that time, the present author discussed the matter of slope-less flat roofs with the aid of a simple model, but applied a rather complicated model for sloping roofs. Lecturers need a simple instructional model that covers all possible roof systems, with or without slope, both very flexible and rather stiff ones. Such an instructional model can be devised as an extension of the simple model in the 2006 issue. In addition, it is discussed how to prevent unstable computations for roof structures not failing by strength, but by stability. The proposed solution is to perform the analysis volume-controlled instead of control by water level.

Key words: Education, instructional model, ponding, water accumulation, roof safety

1 Introduction

Ponding on flexible lightweight flat roofs may occur after a heavy downpour in combination with failing emergency drain systems. Wijte (2006) explained that the phenomenon particularly applies to roofs with low stiffness, moderate permanent loading and large roof areas. Severe structural damage occurs when the regular rain water drainage system is failing or is not adequate for the structure and the emergency drain system does not take over and function as intended. It is also necessary that much rain falls in a short time. Wijte also introduced the notion of *water raising capacity* for a class of roofs which are very vulnerable to ponding.

Structural engineers are used to designing structures for conservative loads such as dead weight, wind and snow, which are not dependent on the deformation of the structure.

However, the case of rainwater load is *not* conservative; the water load does not depend on rainfall alone, but also on the roof deformations. This has embarrassed many a structural designer. *Bouwen met Staal*, the Dutch organisation for the steel construction industry, distributed a special publication *Technisch Dossier* (2006) on the subject of ponding ("water accumulation") on flat roofs. And the Dutch Standard Organization NEN issued a design directive NPR 6703 (2006), an attachment to the norm NEN 6702 (2006). The present author addressed the subject in Heron (2006), ranging from a simple model to an advanced one, all material intended for analysis by hand.

Now, some 15 years later, flat, lightweight roofs still collapse, albeit in substantially smaller annual numbers. It is to be expected that attention to the subject may again increase in time due to climate change and energy transition. Two new phenomena are relevant: (i) short showers are expected to have increased intensity, and (ii) roofs will more often carry solar panels for energy generation. Insurance companies will require sound analyses, if the lifetime load of a roof changes.

The question arises whether the average structural engineer is really aware of the seriousness of the tricky load case of sudden downpours. Notwithstanding all efforts, listed above, the subject appears to be complex and non-linear, and still demands demystification. The intention of the present paper is to discuss the subject with a minimum of math and mechanics and a maximum of understanding. The number of pictures will be maximized at the cost of complexity. It is hoped that professional lecturers may benefit in the classroom from the plain, uncomplicated instructional model that is demonstrated. This paper is certainly also aimed at software providers who have to explain the matter to their clients. Providers will receive hints on how to adapt their packages.

We will first briefly call to mind the simplest model in Heron (2006). The intention is not to really change that model, but to deal with it in a different way. This limited recapitulation only applies to flat roofs without slope, after which the alternate look will be effected for both slope-less and sloping roofs.

2 Short recapitulation

In the 2006 paper, the flat roof is supposed to exist of simply-supported primary members (beams, trusses) at centre-to-centre distance a. For the present, secondary beams and profiled steel sheeting are considered sufficiently stiff to ignore their deformation. The primary beams have bending stiffness EI and span l. It is supposed that camber cancels out the deflection due to permanent loading. The considered load on the roof is rainwater, only. For the moment, we assume that the primary beams run horizontally and the water is flowing in a stationary state through emergency drainage outlets at the vertical edges of the roof. The water surface is at a distance d above the roof supports. Hereafter, we will refer to d as *water level*. If the beam has infinite bending stiffness, the water depth is d over the whole span, so the load is homogeneously distributed. If the beam has a finite bending stiffness, the load varies over length, because of beam deflection, the largest value occurring at mid-span.

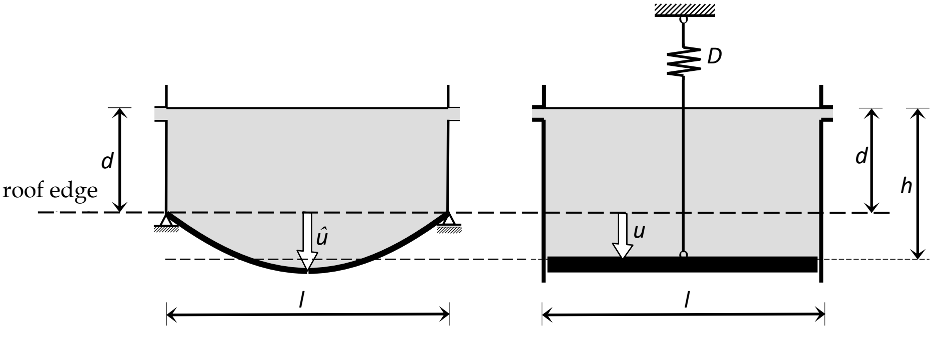

Figure 1. Modelling of the real beam (left-hand picture) to the piston-spring model (right-hand picture)

In the 2006 paper, the beam and water loading were replaced by a piston-spring model, in which the piston can slide frictionless in a rectangular tube. Figure 1 depicts how the real beam (left-hand picture) is modelled (right-hand picture). The weightless piston is supposed to be of infinitely stiff material. The length of the piston is equal to the span l, and its width to the centre-to-centre distance a of the primary beams. Therefore, the area of the piston is $a\,l$. In empty state, the emergency discharge outlet is at a distance d above the piston, and the position of the piston surface is at roof edge level. In the 2006 paper, a stationary state is considered of continuous rain which fills the tube above the piston, while water flows through the outlet. Because of the weight of water, the real beam will displace with a maximum deflection $\hat{u}$, and the piston in the model over an equivalent distance *u*. The water column on the piston has the height *h*, the sum of *d* and *u*. Stated popularly, the displacement *u* can be considered the 'average' deflection of the beam. We arrived in 2006 at the relation:

$$u = 0.8\,\hat{u} \tag{1}$$

For the derivation we refer to Appendix A. An infinitely stiff spring will lead to $u = 0$, so *d* will be equal to *h*, but in practice roof stiffness is never infinitely large, so the height *h* of the water column on the piston becomes larger than the water level *d*. *The two quantities d and h will play a leading role in the development of the instructional model.*

Two parameters *D* and *W* appear to be important in the 2006-paper, of which *D* is the *stiffness* of the spring and *W* is the *weight* of a water column on the piston of unit height. For *D* and *W* the following formulas were derived:

$$D = 96\frac{EI}{l^3}\,; \quad W = \gamma\, a\, l \tag{2}$$

where γ is the specific weight of water. We refer to Appendix A again. The parameters *D* and *W* have the same stiffness dimension kN/m. It appeared useful to define a *stiffness factor n*, being the quotient of *D* and *W*:

$$n = \frac{D}{W} \tag{3}$$

This ratio does not have a unit, it is dimension free. In the classical ponding theory, a slightly different definition of *n* is often used. That definition reads $n = EI\,/\,EI_{cr}$. If we use $EI_{cr} = \frac{1}{96}\gamma a l^4$, see Appendix A, this alternate definition results in the same value of *n*.

In 2006, the water accumulation problem was posed as follows:
Choose a drain at some height d above the beam supports, so select a water level d, and calculate which deformation u of the piston will result, or rather, which water column h, being the sum of d and u, will determine the load on the piston.

The analysis proceeds as follows. The water load F_{water} (= $W h$) must equalize the spring reaction force F_{spring} (= Du):

$$Du = W h \tag{4}$$

yielding, in view of Equation 3, the relation between the piston displacement *u* and the water column *h*:

$$n\,u = h \tag{5}$$

In Figure 1 we read $u = h - d$. Substitution of this relation in Equation 5 leads to the relation $n\,(h - d) = h$. After reordering, we obtain a relation between the water column *h* and the water level *d*, depending on *n*:

$$h = \frac{n}{n-1} d \tag{6}$$

In the classic ponding theory for flat roofs, this relation is a main result, and the magnification factor n/(n - 1) plays an important role in classic ponding studies.

In words, Equation 6 states that a water level *d* at the edge of the roof is magnified to a water column *h* due to accumulated water in the deflected roof structure. Many a code refers to this magnification factor.

3 Interpretation for flat roofs without slope

Figure 2 depicts the relation between the water surface *d* and water column *h* for different values of the stiffness ratio *n*. The figure shows graphs for *n*-values larger and smaller than unity. Note that *n*-values larger than 1 result in a positive *d*, so water levels above the roof edge, and *n*-values smaller than 1 in a negative *d*, so water levels below the roof edge. A familiar interpretation is that *h* becomes infinitely large for $n = 1$, and it is commonly concluded that no horizontal flat roofs can be built for $n < 1$. In this chapter, discussion of the graphs is restricted to $n > 1$.

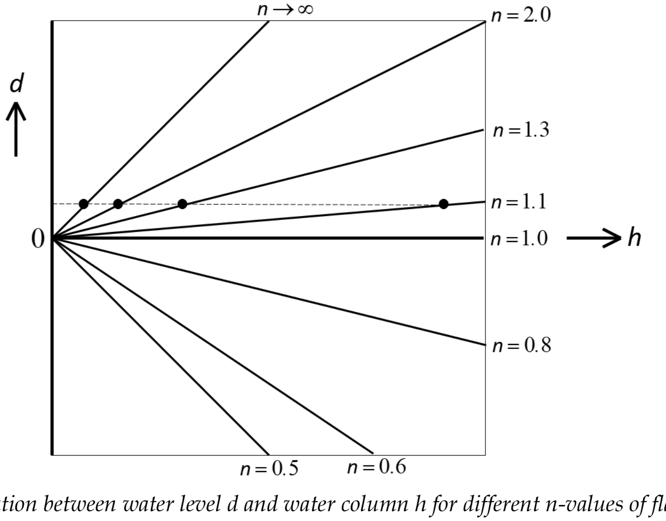

Figure 2. Relation between water level d and water column h for different n-values of flat roofs

The impact of Figure 2 is better understood if a fixed value *d* is chosen (said in another way, a fixed position of the discharge is supposed) and we examine how the piston-spring model predicts the water column *h* for different *n*-values. This is done in Figure 3. The clear message is that the piston carries a water column *h*, which is larger than expected by the water level *d*, and that the water column *h* rapidly becomes higher for decreasing *n*-values. Four cases with the same *d* result in huge differences of water volume.

In real practice, ponding analyses are performed with computer programs, where the value of *h* is obtained in an iterative procedure (in fact the deflection $\hat{u}$). If the stiffness ratio *n* is close to 1, a huge number of iterations may be needed, or no solution is obtained due to divergence problems.

4 Alternate interpretation for slope-less flat roofs

An alternate interpretation is obtained if we use the described model in a slightly different way. Again, Figure 1 is the starting point, but now the water accumulation problem is posed in a reversed way.

Above, we fixed d and calculated the water column h, but now we fix h and calculate d.

Fixing *h* means that we consider a given volume of water. Now, Equation 6 appears in reciprocal kind:

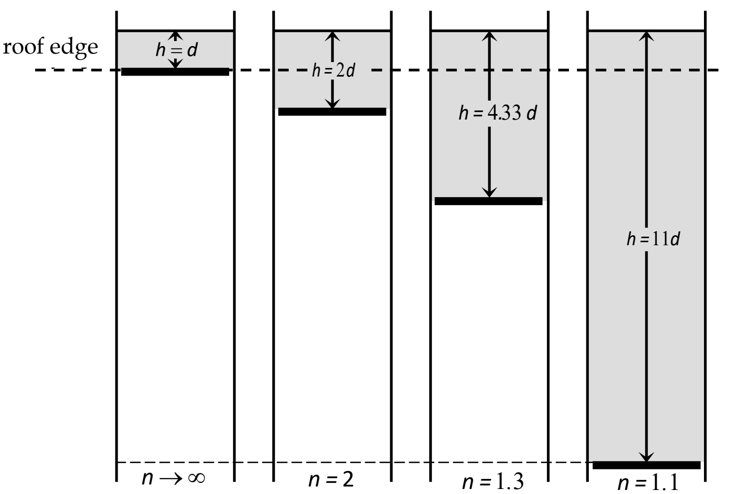

Figure 3. Water column h for decreasing stiffness ratios n (same discharge position)

$$d = \frac{n-1}{n} h \tag{7}$$

Figure 4 depicts the relation between h and d for different n-values. Curves for $n < 0.5$ are left out again, because they will not easily occur in practice. In the plot, again four combinations of d and h are selected by a dot, now all with the same water column h. The choice of a fixed h means that the four dots represent cases of different n-value, but all with the *same volume of water*.

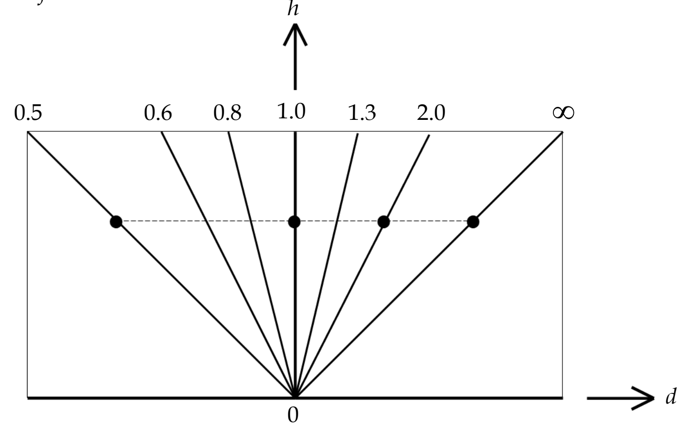

Figure 4. Relation between water column h and water level d for different n-values

Figure 5 depicts which position the piston takes in the four chosen cases. Now the message is that the piston (and the water column h) displaces further down for smaller n-values, and that states are possible where the water level becomes equal to or even below the roof edge. The water level stays above roof edge if $n > 1$, will be at roof edge for $n = 1$, and goes down the roof edge for $n < 1$. And, importantly, all four demonstrated cases are *stable equilibrium states*. The message is: Choosing a value of h, or in other words *controlling by*

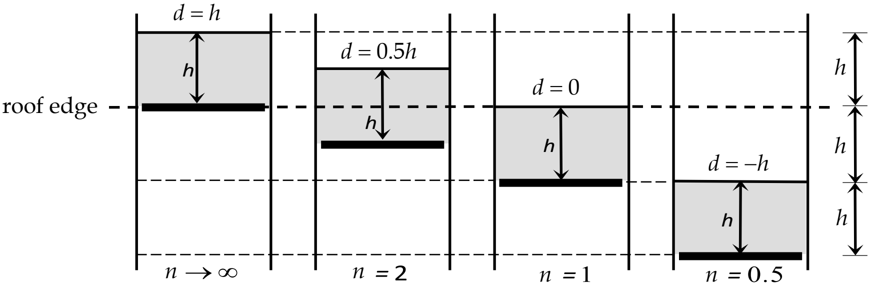

Figure 5. Water surface d for decreasing stiffness ratios n (same water column h)

volume, always yields a stable analysis. If applied in computer programs, no divergence will ever occur.

In the first paragraph of Section 3, we recalled the common conclusion that no horizontal flat roofs can be built for $n < 1$. Returning back to that conclusion, strictly speaking, the statement is not the whole truth. On the basis of the plots in Figure 5, one can only conclude that for $n < 1$ the water surface will become lower than the roof edge, since d is negative. Flow of water through an emergency outlet above the support is no longer possible, but due to the deflection of the primary roof beams, water will still be stored. If the regular water drainage option could be installed halfway along the roof span, the location where the deflection in the real beam is maximal, a well-functioning roof system might be obtained. The same applies for the emergency outlet system. However, such illogical solutions for extremely flexible roofs would require well-organized inspections on a regular basis and at well-chosen time intervals. There are more adequate solutions in practise, as we are to show hereafter. That solution is the application of a sloping roof.

5 Sloping roof

Having studied flat roofs without slope, a simple model for sloping roofs is close at hand to examine the occurrence of n-values below 1. The same primary beams, secondary beams and steel sheeting are considered, and the same supports. The only difference is a moderate slope of a small percentage. Structural designers should notice that the definition of the n-value does not change if roofs have such a slope. It is calculated in the same way as for slope-less flat roofs. Equations 2 and 3 remain in full force.

Similar to the flat roof without slope, the span is l, the centre-to-centre distance of the primary beams is a, and the considered roof area is $a\, l$. The slope is introduced by lifting the right-hand support over the distance d_0 with respect to the left-hand support. And again, the deflection due to permanent load is supposed to be annulled by an appropriate camber.

The spring-piston model for the sloping roof is depicted in Figure 6. For drawing reasons, the vertical scale in the figure is exaggerated. In reality the slope angle is small, choosing d_0 only a few percent of the span l, in The Netherlands at minimum 1.6%. In dry state, the roof beam in Figure 6 is the inclined straight dashed line from the lower to the upper support. The only difference with the model for the horizontal flat roof is the slight inclination of the piston. The way in which the slope is realized, implies that the left-hand part of the roof will start to fill with water. Therefore, the left-hand end of the piston is now called the roof edge.

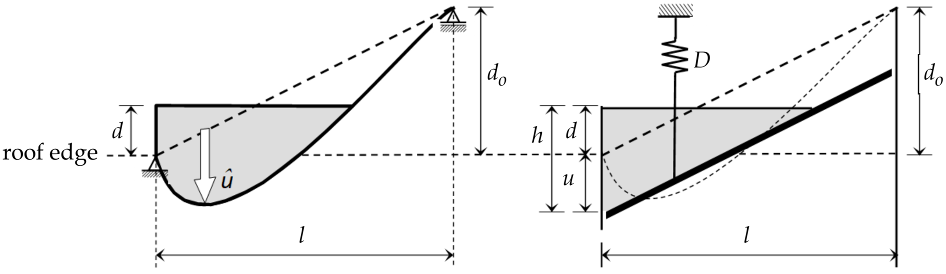

Figure 6. Modelling of the real sloping beam (left-hand picture) to the inclined piston-spring model (right-hand picture)

If the water level has risen to d, the beam will be partly covered by water, will deflect, and due to the deflection will store additional water. In the model, the inclined piston will move downwards parallel to its original dry position (dashed line), and tensions the spring. Similar to the flat roof without slope, the maximum deflection of the real beam is $\hat{u}$ and the piston displacement is u, see Figure 6.

In Section 4 we had chosen a fixed water column h and derived the value of d. Doing so, we followed a *volume-controlled procedure*, because the water surface had a constant value $a\,l$ independent of the water level. This does not hold true anymore for the inclined piston. Yet, we can choose a volume-controlled procedure, which is explained on the basis of Figure 7.

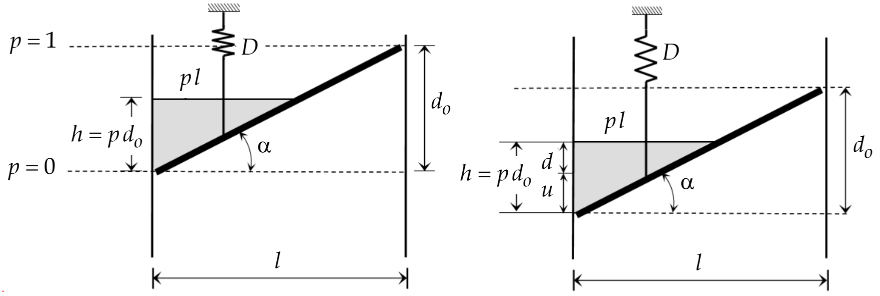

Figure 7. Introduction of dimensionless parameter p

We consider a triangular volume of water with height h as depicted in the left-hand part of the figure, but must now account for the fact that the height of the water varies over the roof. Note that the water column h is a measure for the water volume V, but the relation will not be a constant factor anymore.

It is convenient to introduce a dimensionless parameter p, which is zero at the lower end of the piston and takes the value 1 at the upper end. Then, for values $0 \le h \le d_o$, the height h is $p\,d_o$ and the length of the water surface is $p\,l$. Hence, because of the centre-to-centre distance a of the primary beams, the area of the water surface is $p\,a\,l$. With this information, we calculate the piston displacement for the considered water volume. The volume of the water is $V = ½ \cdot p\,d_o \cdot pal$ and the weight is $F_{water} = \gamma\left(½ \cdot p\,d_o \cdot pal\right)$. In view of $W = \gamma\, a\, l$ we arrive at the water weight $½\, p^2 d_o W$. The resistance F_{spring} to this weight is Du, so the equilibrium equation becomes:

$$Du = \tfrac{1}{2} p^2 d_o W \tag{8}$$

After division by D, accounting for $n = D/W$, we can rewrite the equilibrium equation as:

$$u = \frac{p^2}{2n} d_o \tag{9}$$

Again, $u = h - d$, in which we substitute $h = p\,d_o$. The result is a relation between u and d: $u = p\,d_o - d$. Substituting this expression of u in Equation 9, and solving for d, leads to the end result:

$$d = (p - \frac{p^2}{2n}) d_o \qquad (0 < p \le 1) \tag{10}$$

Each n-value in Equation 10 yields a parabolic function of d in p. These functions are plotted in Figure 8, where the horizontal coordinate is $p = h / d_o$. For $0 \le h / d_o \le 1$ the plots represent water columns smaller than d_o; for $h / d_o > 1$, the plots represent water columns higher than d_o. In fact, the part larger than d_o is based on Equation 7, so then the multiplication factor $(n - 1)/n$ applies. The figure only shows plots for n-values smaller than or equal to 1.00. Each of the plots has a limit value $\hat{d}$, for which – as previously stated − Wijte (2006) introduced the term *water raising capacity*. These extremes occur for the value $p = n$, so for the value of h:

$$h = n\, d_o \quad (n \le 1) \tag{11}$$

If n is close to 1.00, the limit point is in the immediate neighbourhood of the right-hand

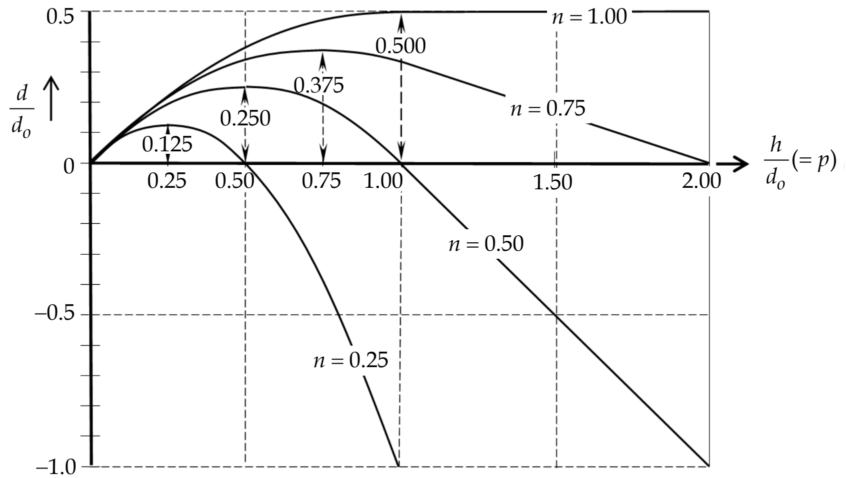

Figure 8. Relation between water level d and the water column h for low n-values of sloping roofs

upper support. For n-values approaching zero, the limit point is close to the left-hand lower support. The value of the extreme, the water raising capacity, is:

$$\hat{d} = \tfrac{1}{2} n\, d_o \quad (n \leq 1) \tag{12}$$

The phenomenon of a limit water level $\hat{d}$ only occurs if $n \leq 1$. For these n-values, the water level cannot surpass the limit in any rain shower. And, because of Equation 12, for no stiffness constant $n \leq 1$ the raising capacity $\hat{d}$ can become larger than ½ d_o. Stated in words:

When $n \leq 1$, the water level can never raise higher than halfway along the slope.

The volume V at which the limit point occurs, is in practical cases still small, such that the steel members are far from yielding or local instability problems. The structure does not fail due to lack of *strength* but due to loss of global *stability*. This observation was reason for the Dutch code writers to use in NEN 6702 and NPR 6703 the name $d_{hw,stab}$ where here $\hat{d}$ is used.

Practically all software packages that are able to run ponding analyses require specification of the water level d and compute in an iterative way what volume of water will be stored on the roof, and what steel stresses occur. If $n < 1$, programs cannot converge for specified d-values larger than the raising capacity $\hat{d}$. No equilibrium solution does exist in those cases.

It is instructive to visualize which position of the water column in the piston-spring model is associated with the plots of Figure 8. For this purpose, Figure 9 is added, where we depict for graph n = 0.5 the position of the water level and the position of the piston for six values of h / d_o (i.e. six volume values). The first four plotted bold bars hold for the domain in which the water column at the roof edge is smaller than or equal to d_o. The last two hold for the domain where the water column is larger than d_o. The piston is below the roof edge in all six cases, and the water surface will also be below when the water column h is larger than d_o. After passing the water raising capacity, all plots have a descending branch and somewhere on this branch the water load has increased such that the structure, as yet, will fail by strength.

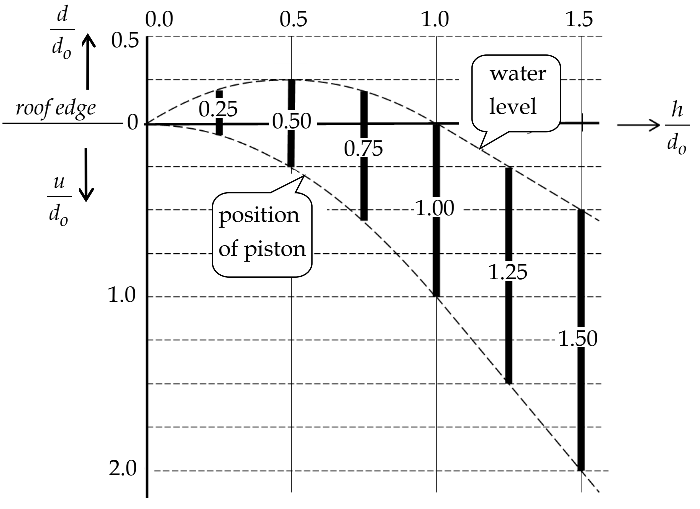

Figure 9. Position of water level and piston for n = 0.5 and six h-values. The number in the bold bars is the value of h / d_o.

5.1 *Extension to all n-values*

Until now we have restricted the discussion of sloping roofs to n-values smaller than 1. This is not necessary, because these roofs will always function for n-values larger than 1, even better the larger n becomes. For this reason, Figure 8 is repeated in Figure 10 with the extension of plots for $n > 1$. For negative values of d / d_o the domain is shortened to half. For $h < d_o$ the water level is between the lower end and the upper end of the piston, and the curves are parabolic. For $h > d_o$ the water level is above the upper end, and the relation between d and h becomes linear. Then, the gradient conforms with the multiplication factor $(n - 1)/n$ for slope-less roofs, according to Equation 7.

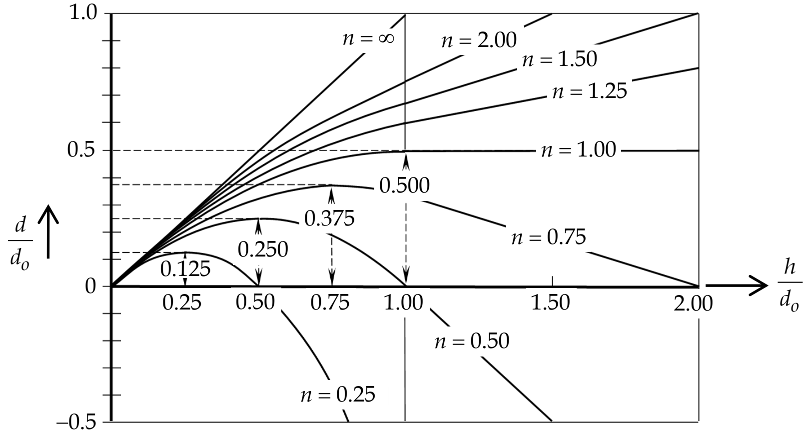

Figure 10. Repetition of Figure 8, completed with graphs for n > 1

6 Volume control

Similarly to the findings for slope-less flat roofs, the procedure for sloping roofs on the basis of a chosen volume always yields a stable solution; no danger of divergence exists any more. We may prefer to present Figure 8 in a slightly different way, knowing that the variable h at the horizontal coordinate axis is a measure for the volume V. In Section 5, we obtained the expression $V = \frac{1}{2} \cdot pd_o \cdot pal$. If we introduce the reference volume

$$V_o = a\,l\,d_o \tag{13}$$

the expression $V = \frac{1}{2} \cdot pd_o \cdot pal$ is transferred into:

$$\frac{V}{V_o} = \frac{1}{2}p^2 \qquad (0 < p < 1) \tag{14}$$

Hence, the horizontal coordinate $p = h / d_o$ in Figure 8 can be replaced by V / V_o, which will change the scale of the graphs in a horizontal direction. As a consequence, the shape of the graphs will change. Figure 11 shows the plots for different n-values. The left-hand half of the figure ($0 \le V / V_o \le 0.5$) holds for volumes at which the piston is partly or just completely filled to its upper end. In the right-hand part ($0.5 \le V / V_o \le 1$) the upper piston end is always fully submerged.

Again, to support proper understanding, the plot for n = 0.50 in Figure 11 is clarified in Figure 12. The position of the piston and the water filling are depicted for four different volumes V / V_o: 0, 0.125, 0.5 and 1, which is a selection from the six cases of Figure 9. In the

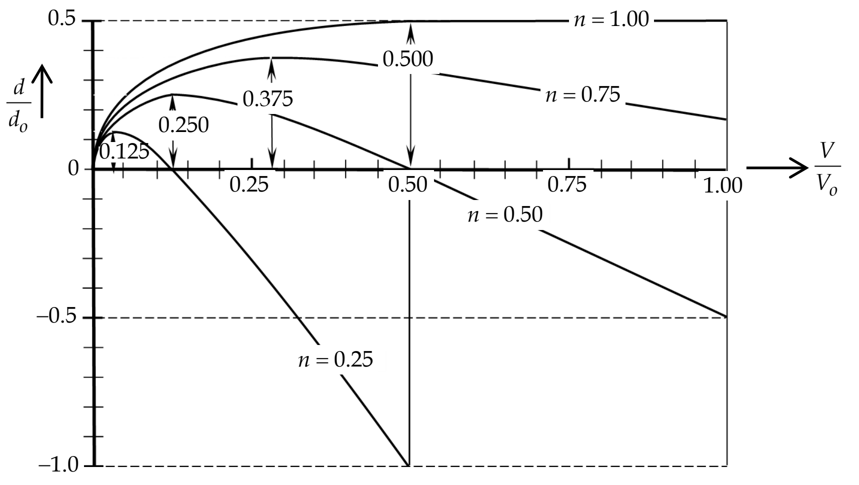

Figure 11. Relation between d and the volume V of sloping roofs for different values n ≤ 1

first plot, the piston carries no water at all and does not displace, in the second one the piston is filled to half the height of the slope and has a downward displacement 0.25 d_o, in the third one the piston is filled up to its upper end and the piston displaces downwards d_o below the roof edge. In the fourth plot the upper end of the piston is completely submerged and the displacement of the piston below the roof edge is 2 d_o.

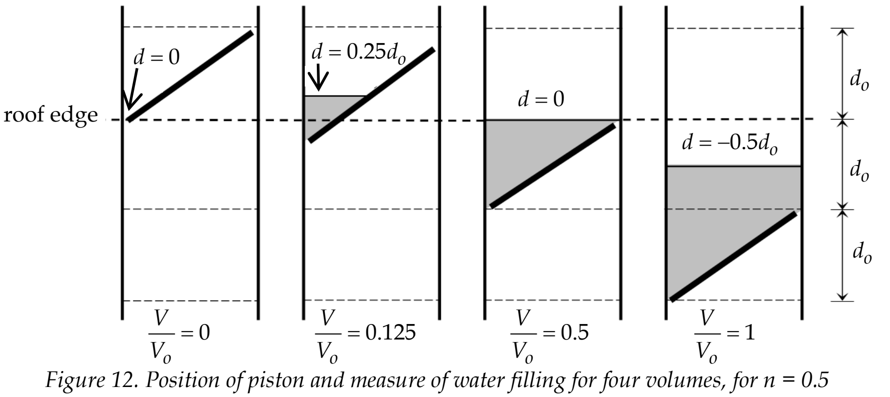

Figure 12. Position of piston and measure of water filling for four volumes, for n = 0.5

6.1 Converging computer simulations

In the same way as Figure 8 was extended to Figure 10, extension of Figure 11 to Figure 13 can be made. It is an important message of the figures that computer simulations will always converge if a volume V (value on the horizontal axis) is specified, and the corresponding water level d (value on the vertical axis) is computed.

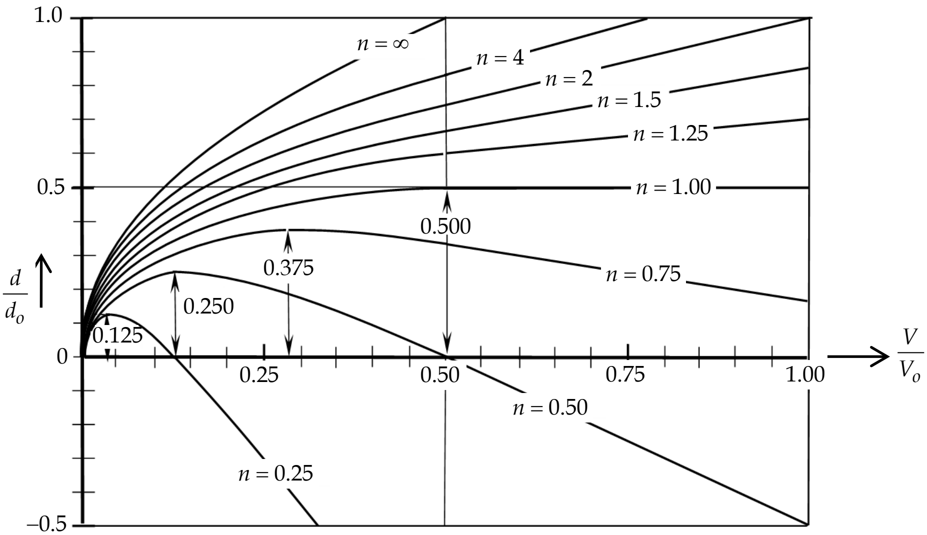

Figure 13. Repetition of Figure 11, complete with graphs for n > 1

In reality, there is a small difference between real roof beams and the piston-spring model. In the model the water volume stays unchanged, when the piston displaces, but for real roof beams the stored water volume may not stay at the specified value if the beam deflection increases. Therefore, in each volume step a refining iterative procedure is necessary to keep the volume unchanged, but success is guaranteed. Diverging analyses belong to the past.

An otherwise good procedure avoids the need for refinement within a volume step. Then we accept the outcome of the first result in a volume step, and save the obtained (V, d) couple. We increase the obtained V by the intended ΔV and proceed immediately to the analysis for the next volume step, obtaining a new (V, d) couple. Thus, we can still plot a graph in which the water level d is a function of volume V, the only difference being that no equal steps of volume are used. We run the program as often as the specified number of volume steps. The pertinent goal is anyhow reached: if $n > 1$, we obtain the specified d; if $n < 1$, the water raising capacity $\hat{d}$. Again, no more diverging analysis.

6.2 *Comparison with computer analyses*

Figure 13 has been obtained on the basis of the simple inclined piston-spring model depicted in Figure 6. In Appendix B the figure is repeated and compared with the output of computational analysis. From that comparison we conclude, that the difference in the water raising capacity between the plain model and the computations is small if n keeps close to 1. More importantly, the water raising capacity in the computations is larger than in the plain model for all n-values, so the model produces safe results.

7 Combination of dead load and water live load

Until now, we have supposed that deformation due to permanent loads is compensated by camber. If camber is not applied, the structure has an initial deformation, of which we call the displacement mid-span $\hat{u}_i$. The initial displacement in the piston-spring model is indicated by u_i , and is related to $\hat{u}_i$ by $u_i = 0.8\ \hat{u}_i$. We will discuss how the initial deformation changes the analysis, both for flat roofs without and with slope.

7.1 *Slope-less flat roofs*

Figure 14 shows in what way the spring-piston model is adapted. In dry state the position of the piston is u_i lower than the supports of the beam. If a water column h is chosen, the piston moves down over a distance u. The water column h is composed of three parts, from bottom to water surface: u, u_i and d. Herein, the water level d is the distance from the roof edge to the water surface, as we defined before. Equation 4 still holds true: $D\ u = W h$.

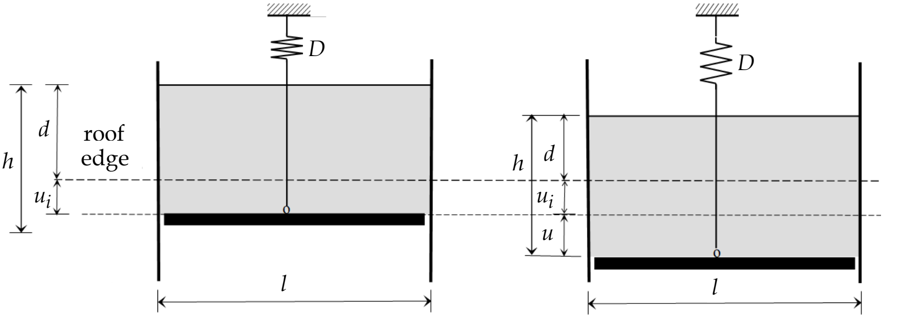

Figure 14. Extension of model for slope-less flat roof by initial displacement u_i due to permanent load. Left: Undeformed. Right: Deformed. Left and Right: Same water column h.

Instead of $u = h - d$ we now substitute $u = h - (d + u_i)$. Accounting again for $n = D / W$, the relation $D\,u = W\,h$ transforms into:

$$d + u_i = \frac{n-1}{n} h \tag{15}$$

If compared with Equation 5 for beams with camber, we just replaced d with $d + u_i$. That is to say: for a specified water column h, the water level d will become u_i lower; the same holds for the position of the piston. Therefore, in Figure 2 the origin O shifts over a distance u_i upward, and in Figure 4 to the right.

7.2 *Sloping flat roof*

Figure 15 depicts in which way the sloping spring-piston model is adapted if an initial displacement u_i is involved. Accounting for u_i in the derivation of Section 5 yields the same conclusion as was obtained for slope-less roofs. We must simply replace d with $d + u_i$ in Equation 10. In Figures 8 to 11 and Figure 13 the origin must shift upwards, and the value of $\hat{d}$ in Equation 12 must be diminished by u_i .

8 Composed roofs

In Section 2, for the time being, secondary beams and profiled steel sheeting were considered sufficiently stiff in order to ignore their deformation. In reality, their deformation may contribute noticeably to the deflection of the roof structure, at least to some extent the secondary beams. The effect of deformable secondary beams an d steel sheeting is easily accounted for by adapting the value of n as discussed in the special *HERON* edition on ponding, see Blaauwendraad (2006), at least for regular rectangular

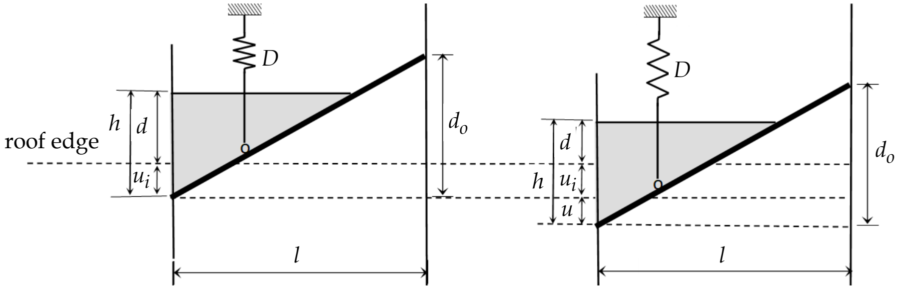

Figure 15. Extension of model for sloping flat roofs by initial displacement u_i due to permanent load Left: Spring not yet deformed. Right: Spring deformed. Left and Right: Same water level h.

roof plans. Then, the integral model consists of three different piston-spring models in line, of which the three springs are chained in series, see Figure 16. The pistons of the primary and secondary members are perforated in order to have the water pass to the steel sheeting piston. In that way, the sheeting carries all the water load and the three springs form a chain. The three piston-spring models have each their own n-value (n_p , n_s and n_{sh} , respectively). The integral value n is computed with the formula:

$$\frac{1}{n}=\frac{1}{n_p}+\frac{1}{n_s}+\frac{1}{n_{sh}} \tag{16}$$

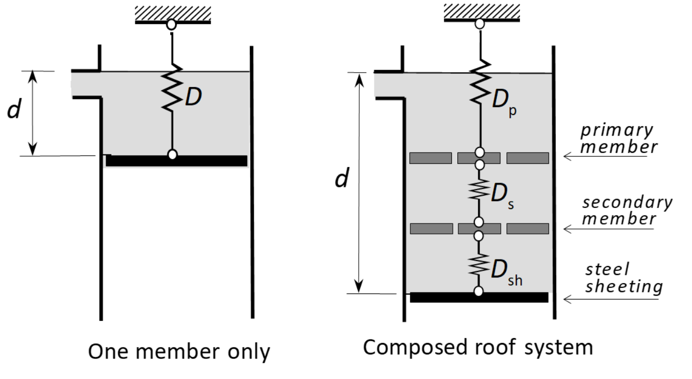

Figure 16. Composed model of primary members, secondary members and profiled steel sheeting

9 Discussion

9.1 Failure by strength or stability

For students and structural designers, the advantage of the model is its simplicity. High-school mathematics is sufficient to understand what is happening when sloping roofs have n-values below unity. The model is safe, as it is conservative. Particularly, the prediction of the water raising capacity $\hat{d}$ is important. For values $n > 1$, the raising capacity is not relevant, because d will always increase for growing volume V. So, the water level can keep raising until structural strength is surpassed. This failure type is *strength-controlled*.

On the contrary, for n-values below unity, the structural failure is *stability-controlled*. When the water volume approaches the value at which the water level reaches the raising capacity, stresses are still moderate. If the volume V keeps growing, after passing the limit point, the water level will go down until the water volume surpasses the strength capacity of the roof structure, inducing failure. As was seen, the water level can even end up below the roof edge.

The message for the structural engineer is to be suspicious of extremely slender roofs.

If water can pour in freely, the structure will definitely fail after passing the limit point. In case such roofs are not to be avoided, the emergency outlets should be positioned low at the roof edge. They also must have enough capacity, in order to have the water flowing freely. In critical cases − the roof edges not being sufficiently high − the emergency outlets have to be installed in the roof surface itself. This must be done at some distance from the roof edge, where the deflection is expected to be maximal. Again, such flexible roofs would require well-organized inspections on a regular basis and at well-chosen time intervals.

9.2 ***Model factor***

In the Dutch Code NEN 6702 (2006), load factors for permanent load and variable load and a material factor warrant the safety of the structure. Naturally, this typically refers to structures failing by strength. However, load factors and material factors are not a proper way to warrant safety in structures that fail by stability. Then, stress levels are still low. It was a main reason in The Netherlands for introducing an additional factor of safety, called *model factor* γ_M, in the guidelines for practice NPR 6703 (2006), a supplementary of NEN 6702 (2006) with additional and simplified rules.

The model factor is 1.3 for the design of new building structures and 1.1 for existing ones. The factor must be applied to the stiffness of all roof members (primary, secondary and sheeting) dividing the stiffness by the model factor. The goal is to gain an extra safety margin in real life of the roof structure by using a reduced structural stiffness in the design phase. The effect of the model factor for roof structures $n < 1$ can be seen from Equation 12 for the water raising capacity. The capacity is linearly dependent on n, which in its turn is linearly dependent on the bending stiffness EI. So, reducing the bending stiffness by a factor 1.3 directly decreases the computed raising capacity $\hat{d}$ by a factor 1.3. As a reminder, Equation 12 holds for the situation that initial deformation by permanent loads is compensated by a camber. Otherwise, we still must distract u_i .

9.3 ***Combinations of model factor and camber***

In this section we investigate the effect on the water raising capacity for possible combinations of model factor and camber. The model factor is varied, for two camber states, applying it and omitting it. For this purpose, we consider two graphs in Figure 13, one with stiffness ratio n = 1 and one with n = 0.75. We think the value 1 applicable for a real structure, and the value 0.75 the design value prescribed by the model factor of a local code. The ratio of the two n-values is 4/3. Therefore, we consider combinations with the model factor γ_M = 4/3, which is close to the Dutch value 1.3. If camber is applied and no model factor is required, according to graph n = 1 in Figure 13, the water raising capacity is 0.5 d_o . If the model factor 4/3 must be applied (still in combination with camber), according to graph n = 0.75, the raising capacity is 0.375 d_o . The application of the model factor yields a reduction in the water raising capacity of 0.125 d_o .

What reduction will be found if no camber is applied? In that case we must account for the initial displacement u_i due to permanent load. Here, we assume the size of u_i in the order of magnitude l / 250, so u_i = 0.004 l. For the size of the slope, we adopt the Dutch rule that (at minimum) d_o is 1.6% of the span l, so d_o = 0.016 l. Hence, u_i = 0.25 d_o . In Figure 13 this means shifting the horizontal axis over a distance 0.25 upward, which has substantial influence on the water raising capacity. For all graphs $n \leq 1.0$, the capacity is reduced by 0.25 d_o , and for all values $n \leq 0.5$ the water level d even becomes equal or smaller than zero.

For n = 0.75, the water raising capacity reduces from 0.375 to 0.125, and for n = 1 from 0.5 to 0.25. With the model factor γ_M = 4/3 the graph n = 0.75 applies, which has the raising capacity 0.125 d_o .

Table 1. Values $\hat{d}$ / d_o of water raising capacity

		Camber		
		Yes	No	Diff.
	No	0.500	0.250	0.250
Model factor	Yes	0.375	0.125	0.250
	Diff.	0.125	0.125	

The water raising capacities $\hat{d}$ / d_o of the considered combinations are assembled in Table 1. Also the differences are calculated in vertical and horizontal table direction. Four conclusions can be drawn:

1 The water raising capacity of the cases 'no model factor + yes camber' and 'yes model factor + no camber' differs by a factor of 4.

2 The difference in raising capacity is 0.250 d_0 between camber and no camber, regardless of the value of the model factor. The difference is 0.125 d_0 between 'yes model factor' and 'no model factor', regardless of yes or no camber.

3 If safety is expressed in terms of the water raising capacity, the contribution of camber to the safety (0.250 d_0) is two times larger than the contribution of the model factor (0.125 d_0).

4 Structural engineers should not forget to account for initial displacements due to permanent loads in their ponding analyses.

*9.4 **Sensitivity to construction inaccuracies***

One of the reasons to introduce the model factor is to account for construction inaccuracies. Therefore, in the calibration procedure for the correct value of the model factor, also the slope and water level have been treated as stochastic quantities. Van Herwijnen *et al.* (2006) approached the subject in a different way when they discussed the impact of construction inaccuracies on the safety in the case of ponding. In their findings, inaccuracies in the height of the emergency drains and the roof slope exert a large influence on the safety of roof structures for the load case water ponding. They advised to design roof structures with a value $n \geq 1.5$ only, because lower values yield extremely sensitive structures to inaccuracies, regarding of course the height of emergency drains and roof slope.

Regardless the probabilistic calibration studies, the recommendation $n \geq 1.5$ yet deserves sympathy. This advice combats the flaw at source, really excluding roof structures that may fail by stability. The present author has suggested the same solution in the preparation phase of the guideline NPR 6703 (2006). The costs of additional kilograms of steel will be small if compared to the integral building costs. On the contrary, costs of reconstruction of a damaged roof may approach the order of initial building costs. Should not all structural engineers consider it beneath their station to be the designer of such vulnerable roofs? A decision not to allow roof structures $n < 1.5$ may make sense, but raises another problem. Now the structural engineer is faced with the question how to compute in a reliable way the n-value for composed steel roof grids of primary and secondary members covered by steel sheeting. The computation is straight forward for regular rectangular roof plans (refer to Chapter 8), but not for irregular ones. Apart of that, even the analysis for regular plans is an approximation.

9.5 No misunderstanding!

The proposal to operate volume-controlled should not be misunderstood. It is *no plea at all* to start from an ahead specified volume of rainwater on the roof. Instead, we stay thinking in terms of codes that specify a height d of the emergency discharge. These codes expect the structural engineer to prove that the structure is safely able to carry the water load of that level d.

Why yet control by volume? The answer is to avoid a twofold problem. The first is related to roofs that fail by stability. Then, starting with a specified value of d may easily result in *diverging computer runs*. The second problem is – as said above – that the value of n is not known at start of the analysis. So, we do anyhow not know whether the roof structure is of the type failure *by strength* or *by stability*.

To be clear once more, we recommend a computational procedure that delivers a plot, showing how the level d develops if the volume V is increased, starting at zero. If the plot shows an ever rising graph, the specified level d can be reached, and strength-control must be executed. If the plot shows a limit point, the structure is of the type failing by stability, and the structural engineer has the information he needs to yes or no adapt the structure and/or the emergency discharge system.

9.6 Software challenge

Nowadays the average package for structural analysis with ponding functionality is of the type discussed in Section 3. The water level d is input and the accumulated water load (water column h in the model) is output, or rather, the displacements, resulting in information to check strength. This is indeed the way which used to be recommended in the Dutch code NEN 6702 (2006). Such analysis is *water level controlled.*

No very drastic programming intervention is needed to also make a *volume-controlled* version of programs. As stated in Section 9.5, the structural engineer should be able to check if any limit in the water level is due. For that purpose, an analysis has to be performed for a number of volumes, and for each volume the water level must be computed. The computed d-values are presented in a graph as a function of the volume V, which makes it easier for the structural engineer to check the occurrence of a limit point. And if it occurs, the engineer knows what water raising capacity the roof has. If the capacity is not sufficiently large, the structure must be adapted or the emergency drainage system reconsidered.

The proposed procedure requires a number of computations. Different water volumes V must be considered for which d is computed, and each pair (V, d) may need iterations, dependent on the preferred procedure (refer to Section 6.1). The good news is, that the global stiffness matrix in the set of equations does not change. In fact, regardless the preferred procedure (Section 6.1), it is a matter of several load vectors. Hopefully, software providers feel challenged.

9.7 ***Alert function***

The main aim of this *HERON* article is the provision of an instructional model for use in the education of structural engineers. A spin-off, nevertheless, is the application of the simple model in structural practice as a rule of thumb to estimate the water raising capacity, and as an alert for calling in more sophisticated ponding analysis expertise or software where necessary.

Acknowledgement

Water accumulation has been in the centre of interest in the early years of the twenty-first century because of a serious number of roof failures annually. The centre *Bouwen met Staal* (*Building with Steel*), the organisation in The Netherlands for research and promotion in the steel construction industry, and circles in Dutch government have stimulated research and producing guidelines with respect to water accumulation on lightweight steel roofs. The author has enjoyed close engagement with these circles, and appreciated the challenges to study ponding. We are particularly indebted to our former university colleague Ton Vrouwenvelder for reading an early version of the manuscript and his constructive critics and clarifying suggestions.

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Appendix A

A1. Equivalent displacement $u = 0.8\ \hat{u}$

It is assumed that a stable equilibrium state exists with a water level *d* above the support as shown in Figure A1. This water depth is the initial depth before the member starts to bend.

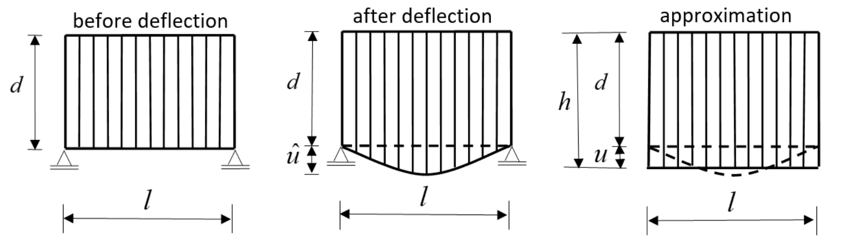


Figure A1. Undeormed beam (left). Deformed (mid). Idealized waterloading (right)

The beam will deflect over a distance $\hat{u}$ and the deflected shape is filled with water as is shown in the middle part of the figure. The water load on the beam now consists of two parts, one homogeneously distributed over the span (initial water depth *d*) and one varying over the span (additional water with maximum depth $\hat{u}$). From here, it is the intention to work with homogeneously distributed loads only, and we substitute the varying part by a statically equivalent constant part with water depth *u*. Statically equivalent means that the same bending moment occurs in the middle of the span. Assuming a sine shape of the deflection and introducing the specific water weight γ results in a half-wave sine-shaped bending diagram with maximum value $M = (\gamma\, a\, l^2)\ \hat{u}/\pi^2$. Herein, *a* is the centre-to-centre distance of the beams. The equivalent homogeneously distributed load causes the maximum bending moment $M = (\gamma\, a\, l^2)\ u/8$. Equating these moments yields, with about 1% accuracy $u = 0.8\ \hat{u}$. The factor 0.8 does not hold true for simply supported members only, but is also applied, with sufficient accuracy, for other support conditions.

A2. Spring stiffness D

The stiffness *D* defines the relationship between the homogeneously distributed total water load *F* on the considered roof (part) and the equivalent displacement *u*:

$$F = D\,u \qquad \text{A(1)}$$

The stiffness D depends on the support conditions. For the time being, we restrict ourselves here to a simply supported member. The relation between F and the displacement u is derived in the following way. It holds $\hat{u} = (5/384)(ql^4/EI)$, where $q = \gamma\,a$, and γ the specific weight of water. Multiplication of both members of this relation by 0.8 yields $u = (1/96)(q\,l^4/EI)$. Next, we replace $q\,l$ by F and rearrange the relation, ending up with $F = 96(EI/l^3)u$. Hence:

$$D = 96\frac{EI}{l^3} \qquad \text{A(2)}$$

A3. Classic stiffness factor n

We repeat the definition of D and W of Section 2:

$$D = 96\frac{EI}{l^3}\,; \qquad W = \gamma\,a\,l \qquad \text{A(3)}$$

In Section 2 the definition of n reads:

$$n = \frac{D}{W} \qquad \text{A(4)}$$

Substitution of Equation A(3) in Equation A(4) leads to:

$$n = \frac{96EI}{\gamma\,a\,l^4} \qquad \text{A(5)}$$

In classic ponding publications a critical bending stiffness EI_{cr} is defined:

$$EI_{cr} = \frac{\gamma\,a\,l^4}{96} \qquad \text{A(6)}$$

Then, an alternate definition of the stiffness ratio n comes into being, because we can write Equation A(5) as:

$$n = \frac{EI}{EI_{cr}} \qquad \text{A(7)}$$

The critical stiffness EI_{cr} is determined by two geometrical data a and l, and the specific weight of water γ. The division of roofs in the types $n > 1$ (failure by strength) and $n < 1$ (failure by stability) is now replaced by the types $EI > EI_{cr}$ and $EI < EI_{cr}$. The classic derivation of n leads to the quotient EI/EI_{cr} and the derivation in this Heron article to D/W, but the result is the same n-value.

However, also another change occurs in the definition of n. The constant 96 in Equation A(6) is π^4 in the classic definition. Then, the n-values of the two definitions are not the same anymore. The difference between the two constants, 96 and 98.7, is almost 3%, and both constants are approximations. The origin of the difference is the different starting point of the definitions. A homogeneously distributed water load is approximated by a sine-shape load in the classic derivation of EI/EI_{cr} , whereas in the actual derivation of D/W a (more or less) sine-shaped water load is replaced by an equivalent homogeneously distributed load.

Equation A(2) can be generalized in order to apply the definition of the stiffness also to other boundary conditions, refer to Blaauwendraad (2006). Then, it holds:

$$D = m \cdot 96 \frac{EI}{l^3} \qquad \text{A(8)}$$

To mention two values of the factor m:

m = 1, for simply supported members.

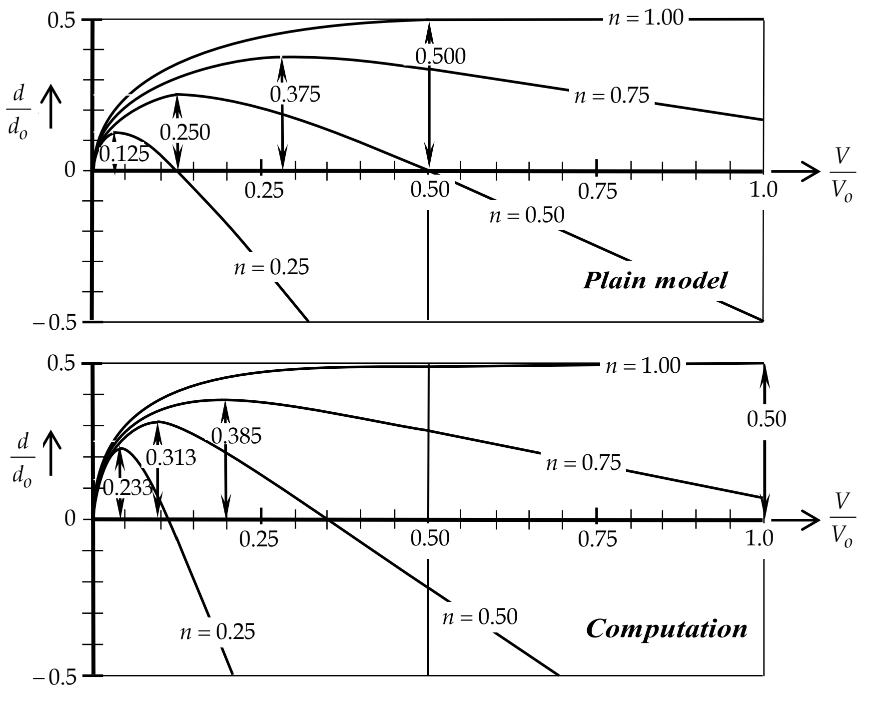
m = 5, for clamp ends of the member.

Appendix B

Comparison of plain model with computer analysis

In Section 6.2 it is stated that the plain model is conservative. The prove of this statement is the subject of this Appendix. In the figure below we reproduce Figure 11 of Section 6.2 as *Plain model*. The corresponding set of graphs from computational analyses are shown as *Computation*.

We performed the analysis for a simply-supported primary beam of constant bending stiffness over the span. The beam is modelled with a series of 15 rigid bars interconnected by lumped rotational springs. We searched the beam stiffness for which the graph has the horizontal line $d / d_o = 0.5$ as an asymptote. We assigned to this graph the stiffness ratio $n = 1$. Thereafter, we multiplied the stiffness by 0.75, 0.50 and 0.25, respectively, to produce the other graphs.



Corresponding graphs in the two figure parts have roughly a similar shape. The correspondence is close for *n*-values near 1.00, and less for $n \leq 0.50$. The water raising capacity of each graph is shown in the picture. Comparison of the two figure parts justifies the conclusion that the raising capacity of the computational analysis is always larger than predicted by the plain model. This supports the statement that the model is conservative.