From quality control to structural reliability: where Bayesian statistics meets risk analysis

R. Caspeele
Ghent University, Belgium
Department of Structural Engineering

Quality inspection plays an important role in the production processes of structural material as well as in design and construction activities, inherently stimulating a higher performance with respect to the investigated activities or properties. In case of the conformity assessment of concrete compressive strength, the concrete strength distribution is filtered due to the rejection or acceptance of certain lots and this filter effect can be quantified using Bayesian updating techniques. A methodology based on numerical integration and Markov Chain Monte Carlo simulations is applied to update prior knowledge with respect to the hyper-parameters of concrete strength distributions, considering the conformity criteria for concrete strength according to EN206-1. Further, as a consequence of this filter effect, conformity control has a positive influence on the reliability of concrete structures and this effect is quantified here by classical FORM calculations. Finally, also the variance reducing effect of conformity control with respect to structural reliability calculations is quantified, illustrating the reducing vulnerability to parameter uncertainties when performing structural reliability calculations.

Key words: Structural reliability, quality control, conformity control, Bayesian updating, concrete

1 Introduction

The material properties are most often a crucial factor in the structural performance of engineering structures. Quality control plays a major role in order to make sure that the production stays more or less stable and the desired properties are maintained in order to assure the material performance in structural engineering applications. Although the basic theoretical framework behind quality control techniques has already a long research history, still new developments are established. These developments in quality control remain a hot topic in scientific research due to its economic importance.

Conformity control is a specific type of quality control often used as a statistical test in normative standards and is commonly used in order to investigate whether a certain inspected lot complies with a predefined (or specified) characteristic X_k of the material property, most often using a decision rule $d(z)$ of the following type [Rackwitz, 1979]:

$$d(z) = \begin{cases} z > a & acceptance \\ z \leq a & rejectance \end{cases} \tag{1}$$

with z a test statistic, e.g. the sample mean $\overline{x}_n$ of n test results, and a an acceptance boundary limit, e.g. in case of concrete strength $a = f_{ck} + \lambda s_n$ with f_{ck} the specified characteristic concrete compressive strength (corresponding to a certain concrete strength class), λ a parameter and s_n the sample standard deviation based on n test results.

As a result of this conformity check, certain inspected lots are accepted and certain lots are rejected. Because of this so-called filtering effect, the original (incoming) distributions of the entire population of the material property can be updated into an outgoing distribution of the accepted inspected lots. In case of concrete strength for example, the filtering effect of the conformity control leads to an increase of the average and a decrease of the standard deviation of the outgoing predictive strength distribution in comparison to the incoming one.

Although these favourable consequences do not form the main objective when designing conformity criteria, they reveal a significant influence on the posterior predictive distribution of material properties and thus should be taken into consideration when updating these distributions. Bayesian statistics can be used in order to update prior distributions of material properties taking into account additional information. Not only direct test results can be used to update these distributions, but also indirect information from e.g. conformity control can be considered when updating the prior knowledge into a posterior belief. Rackwitz [1979, 1983] describes an analytical method in order to evaluate the filter effect of some common (basic) conformity criteria, based on Bayesian statistics. However, in practice also more complex criteria are used, for example with respect to concrete properties in the European Standard EN206-1. In order to evaluate and compare the filtering effect of such more complex conformity criteria, a numerical algorithm was developed by the author in [Caspeele, 2010] and [Caspeele and Taerwe, 2013], based on Bayesian updating techniques and numerical integration, taking into account prior information. Suitable prior information with respect to mechanical properties of currently applied concrete, reinforcing steel and prestressing steel can be found in e.g. [Caspeele and Taerwe, 2011a], [Jacinto et al., 2012], [Wisniewsky et al., 2012]. Further, the developed algorithm uses the operating characteristic of the considered conformity criteria, calculated using numerical Monte Carlo simulations, which also enables to take autocorrelation between consecutive test results into account.

Consequently, these updated distributions can be taken into account when performing structural analysis, especially in case of structural reliability calculations. In case of conformity control, a first analysis with respect to the influence on structural reliability calculations was performed by the author in [Caspeele, 2010], [Caspeele et al., 2013]. From these investigations it was found that conformity control of concrete may positively influence particularly the structural reliability of lightly reinforced concrete members exposed to compression or shear. It appears that it has a minor effect on members with a larger reinforcement ratio or members exposed to bending. Furthermore it enables to ensure a more homogeneous safety level, which is less dependent on parameter uncertainties.

2 Conformity control of material properties

For design and production purposes a material property is characterized by a specified value X_k , which is commonly the 5% fractile of the theoretical distribution of the material property under consideration. In case of concrete for example, the concrete strength is commonly represented by the 5% fractile of the theoretical concrete strength distribution, i.e. the specified characteristic concrete compressive strength f_{ck} (Figure 1).

Design calculations for concrete structures according to the semi-probabilistic safety format of the Eurocodes are often based on this characteristic concrete strength f_{ck} . This implicitly implies that there exists sufficient control on the production process of concrete in order to ensure that – on average – the strength distribution which results from the production process is in accordance with the assumptions on which the design procedure is based.

In practice however, the fraction below the specified value will be smaller or higher than 5%. Designating by θ the fraction of the population below X_k in the offered strength distribution – also called the fraction defectives – it follows that:

$$P\left[X \leq X_k\right] = \theta \tag{2}$$

where X is the material property, considered as a random variable. In case concrete strength is considered, Equation 2 can be rewritten as:

$$P\left[X \leq f_{ck}\right] = \theta \tag{3}$$

which is illustrated in Figure 1.

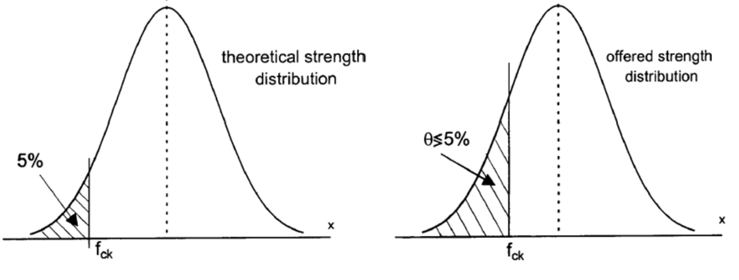

Figure 1: Theoretical and offered strength distributions in case of concrete compressive strength

For an assumed distribution function of the material property under consideration and for a given conformity criterion, one can calculate the probability that an inspection lot, characterized by a fraction defectives θ, is accepted. This probability is called the probability of acceptance and denoted as P_a . The function $P_a(\theta)$ is called the operating characteristic of the criterion and is commonly abbreviated as OC-curve. Realizing that an inspection based on a sample of limited size introduces risks of taking the wrong decision, this OC-curve illustrates the discriminating capacity of a conformity criterion to distinguish “good” from “bad” productions. Analytical formulas in order to calculate these OC-curves in case of some simple conformity criteria are available in [Rackwitz, 1979, 1983] , [Taerwe, 1985, 1988], [Taerwe and Caspeele, 2006], [Caspeele, 2010], [Caspeele and Taerwe, 2011b]. However, for more complex conformity criteria or in case autocorrelation between consecutive test results is deemed important to take into account, numerical Monte Carlo simulations can be used to calculate the OC-curves, which inherently also takes into account any dependency that exists between a set of conformity criteria. More information about the numerical calculation of OC-lines, with or without taking into account autocorrelation, is available in [Taerwe, 1985, 1987, 1988], [Taerwe and Caspeele, 2006], [Caspeele, 2010], [Caspeele and Taerwe, 2011b].

3 The filtering effect of conformity criteria

3.1 ***Principle***

The filter effect of conformity criteria results from the fact that – due to the conformity/non-conformity declaration – inspected lots are accepted or rejected. Because certain inspected lots with deficient quality are rejected from the accepted batches, conformity criteria inherently have a filtering effect on the distribution of the material property under consideration. The average quality of outgoing lots (after acceptance by conformity control) will be higher than the average quality of incoming lots (presented for conformity assessment), i.e. the fraction defectives decreases.

3.2 ***Basic example***

In order to illustrate the basic idea of the filtering effect of conformity criteria, a simple (hypothetical) example is presented in this subsection. Consider for example a control scheme based on attributes which accepts the lot in case no more than c = 1 defective item is found in a sample of n = 30 items and assume that only 2 different fraction defectives θ_1 = 2% and θ_2 = 10% can occur. Considering a binomial distribution for the fraction defectives, the probability of acceptance corresponding to these discrete fraction defectives can be calculated as follows:

$$P_a(\theta_1) = \sum_{i=0}^{c=1} C_{30}^{i} 0.02^{i} 0.98^{30-i} = 0.879 \tag{4}$$

$$P_a(\theta_2) = \sum_{i=0}^{c=1} C_{30}^{i} 0.10^{i} 0.90^{30-i} = 0.184 \tag{5}$$

Assume that both fraction defectives each have an equal prior probability p' of occurring. Bayes' theorem can be used to calculate the posterior probabilities (after acceptance by the conformity inspection), i.e. by taking into account the acceptance probability. The Bayesian updating rule for this case can be written as:

$$P[\theta_i | acceptance] = \frac{P_a(\theta_i)\, p'(\theta_i)}{\sum_j P_a(\theta_j)\, p'(\theta_j)} \tag{6}$$

and the posterior probabilities p'' (conditional on the acceptance of the inspection lot) thus yield:

$$p''(\theta_1) = P[\theta_1 | acceptance] = \frac{0.879 \cdot 0.5}{0.879 \cdot 0.5 + 0.184 \cdot 0.5} = 0.83 \tag{7}$$

$$p''(\theta_2) = P[\theta_2 | acceptance] = \frac{0.184 \cdot 0.5}{0.879 \cdot 0.5 + 0.184 \cdot 0.5} = 0.17 \tag{8}$$

The updating process is illustrated in Figures 2 and 3. Due to the probability of acceptance (associated to a certain conformity control scheme), it is found that in an accepted lot the probability of occurrence of a low quality sample (i.e. with high fraction defectives θ_2) will be lower than that of a high quality sample (i.e. with a low fraction defectives θ_1). Furthermore, the posterior mean fraction defectives is $\theta_1 \cdot p''(\theta_1) + \theta_2 \cdot p''(\theta_2)$ =3.4%, which is lower than the prior mean of 6%. Thus, conformity control shifts the prior distribution towards lower fraction defectives.

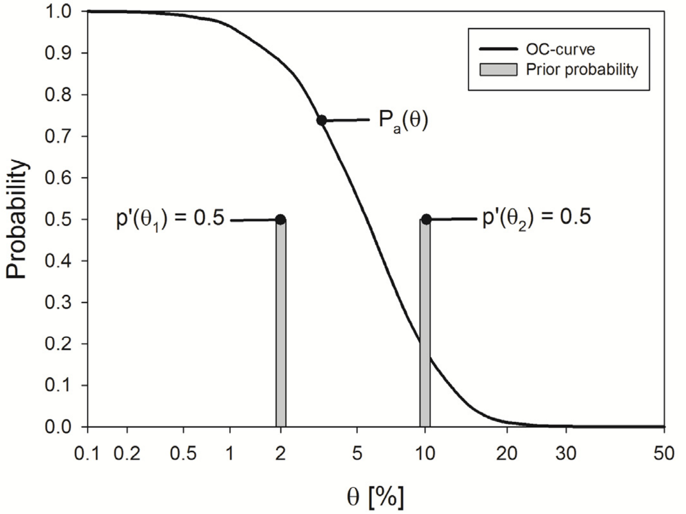

Figure 2: OC-curve and prior probabilities for the basic example

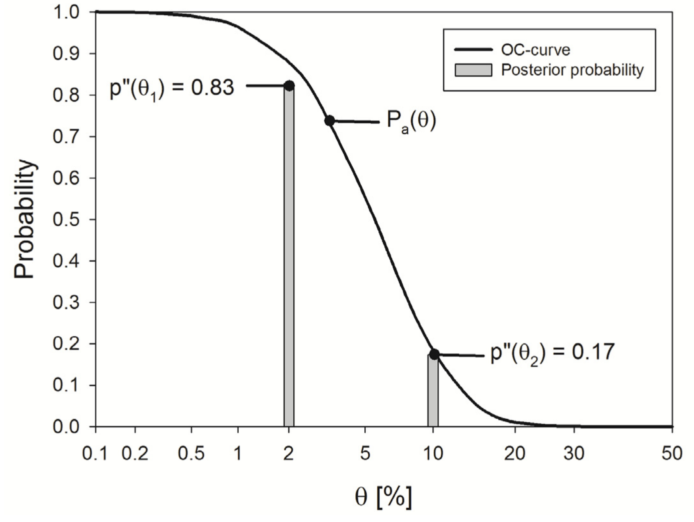

Figure 3: OC-curve and posterior probabilities for the basic example

3.3 *General formulation*

In case of continuous prior distributions, Equation 6 can be rewritten as follows:

$$f_{\Theta,o}(\theta) = \frac{P_a(\theta)\, f_{\Theta,i}(\theta)}{\int P_a(\theta')\, f_{\Theta,i}(\theta')\, d\theta'} \tag{9}$$

with $f_{\Theta,i}(\theta)$ the prior distribution of the fraction defectives in incoming lots (designated '*i*') and $f_{\Theta,o}(\theta)$ the posterior distribution of the fraction defectives in outgoing or accepted lots (designated '*o*') .

However, in case of material properties it is more appropriate to update the parameters of the distribution of the population (i.e. the mean μ and standard deviation σ in case of a normal distribution or $\mu_{\ln X}$ and $\sigma_{\ln X}$ in case of a lognormal distribution, all considered as random variables), because this can then further be used in e.g. structural reliability analyses. In this case, the posterior joint density function of the parameters of the distribution of the material property is given by:

$$f''_{M,\Sigma}(\mu,\sigma) = \frac{P_a(\mu,\sigma)\, f'_{M,\Sigma}(\mu,\sigma)}{\iint P_a(\mu',\sigma')\, f'_{M,\Sigma}(\mu',\sigma')\, d\mu'\, d\sigma'} \tag{10}$$

with $f'_{M,\Sigma}(\mu,\sigma)$ the prior joint density function of μ and σ and $f''_{M,\Sigma}(\mu,\sigma)$ their posterior joint density. Consequently, the posterior predictive distribution of the material property X can then be calculated according to:

$$f''_X(x) = \iint f_X(x|\mu,\sigma) f''_{M,\Sigma}(\mu,\sigma)\, d\mu\, d\sigma \tag{11}$$

Different types of prior information can be used, depending on the objective of the analysis. However, in case these priors are used for updating the distribution of material properties due to a one-sided conformity decision rule (only verifying a lower or upper limit), only informative priors can be used. The use of non-informative priors is not possible, because the shape of the operating characteristic curve in that case does not allow to update uniform distributions. For many practical situations a suitable conjugate prior for the parameters of the distribution of the material property is given by a normal-gamma distribution or a lognormal-gamma distribution [Rackwitz, 1983], [Gelman, 2004]. The latter distribution is defined as:

$$\begin{aligned} & f_{M,\Sigma}(\mu,\sigma|\bar{x}_{ln\,X}, n, s_{ln\,X}, \nu) \\ & = \frac{\sqrt{n}}{\sqrt{2\pi}\mu\sigma_{ln\,X}} exp\left[-\frac{1}{2}\left(\frac{ln\mu - \bar{x}_{ln\,X}}{\sigma_{ln\,X}/\sqrt{n}}\right)^2\right] \frac{\left(\frac{1}{2}\nu\left(s^*/\sigma\right)^2\right)^{\frac{\nu}{2}-1} exp\left(-\frac{1}{2}\nu\left(s^*/\sigma\right)^2\right)}{\Gamma(\nu/2)} \frac{\nu s^{*2}}{\sigma^3} \end{aligned} \tag{12}$$

with

$$s^* \approx \frac{exp(\bar{x}_{ln\,X})}{exp\left(-s_{ln\,X}^2/2\right)}\sqrt{exp\left(s_{ln\,X}^2\right)-1} \tag{13}$$

Prior information can be collected from literature, e.g. in [Rackwitz, 1983],[Caspeele and Taerwe, 2011a] for concrete, in [Jacinto et al., 2012] for reinforcing steel and [Wisniewsky et al., 2012] for prestressing steel. More accurate prior information can also be obtained by Bayesian updating of these priors using additional test results (see e.g. [Raiffa and Schlaifer, 1969], [Box and Tiao, 1973], [Diamantidis et al., 2001]), or they can be estimated using maximum-likelihood estimators [Rackwitz, 1983] or by using a bootstrapping technique in case a limited amount of test date is available [Caspeele, 2010].

From Equation 10 it is observed that the posterior joint probability density function $f''_{M,\Sigma}(\mu,\sigma)$ is proportional to:

$$f''_{M,\Sigma}(\mu,\sigma) \propto P_a(\mu,\sigma)\, f'_{M,\Sigma}(\mu,\sigma) \tag{14}$$

which can be evaluated using numerical integration. A practical calculation algorithm based on numerical integration and Markov Chain Monte Carlo simulations using a cascade Metropolis-Hastings algorithm was previously developed by the author. More details about this methodology are described in [Caspeele, 2010] and [Caspeele and Taerwe, 2013].

In [Caspeele and Taerwe, 2013] the methodology was also applied to quantify the filtering effect of conformity control on the strength distribution of concrete and a few results are repeated in this section to illustrate the phenomenon. As an example of the updating procedure with respect to the parameters of the concrete strength distribution, the prior and posterior joint density function $f'_{M,\Sigma}(\mu,\sigma)$ and $f''_{M,\Sigma}(\mu,\sigma)$ are illustrated in Figure 4 in case of a concrete class C25 (ready-mixed concrete) with suggested prior hyperparameters $\overline{x}'_{\ln X} = 3.65$, $n' = 2$, $s'_{\ln X} = 0.12$ and $\nu' = 4$ according to [Rackwitz, 1983] and conformity criteria for concrete strength according to EN206-1 : 2000. The effect on the predictive concrete distribution is illustrated in Figure 5.

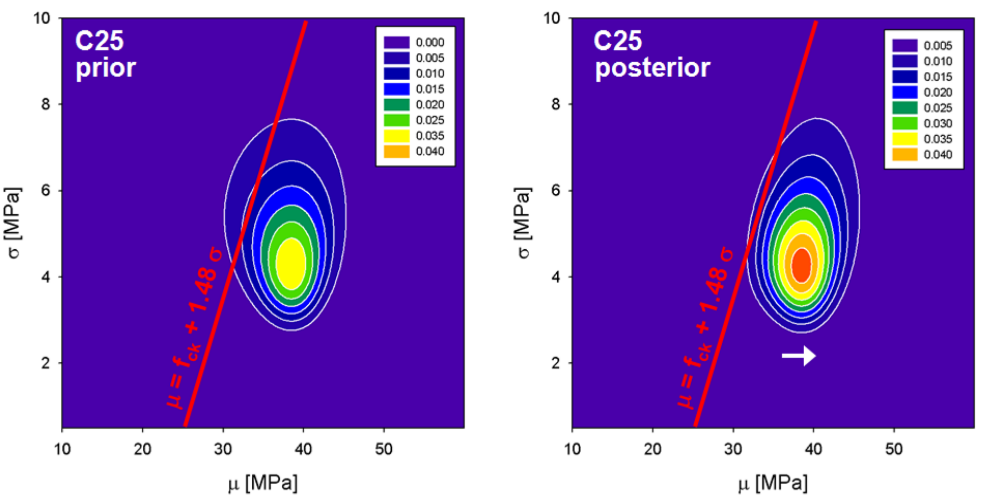

Figure 4: Contour plot of the prior (left) and posterior (right) joint density function of the parameters for the concrete strength distribution corresponding to a concrete class C25 after conformity control based on the conformity criteria under consideration
(A colour figure is available at www.heronjournal.nl.)

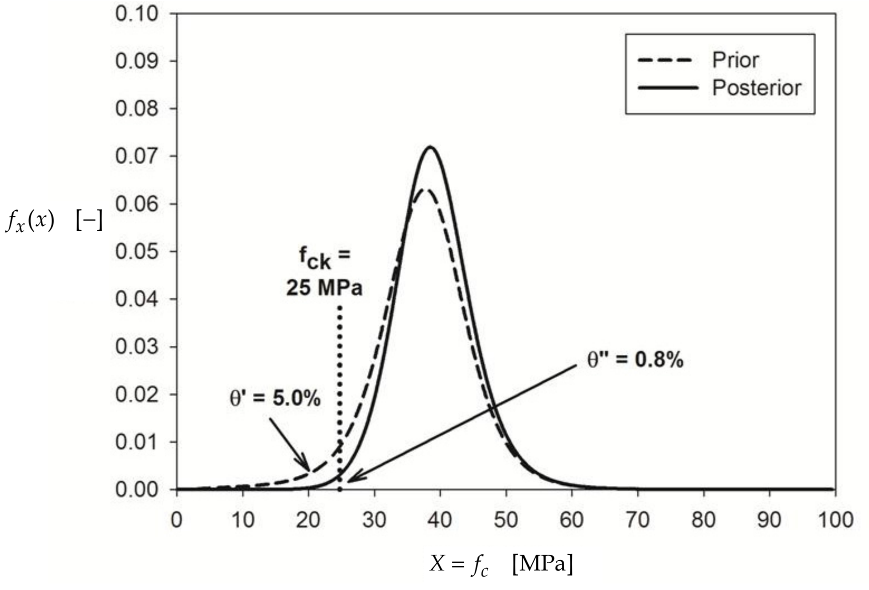

Figure 5: Prior and posterior predictive concrete strength distributions for a C25 concrete class after conformity control based on the conformity criteria under consideration

In comparison to the associated prior distribution, the posterior distribution is shifted slightly towards a higher mean and a smaller standard deviation, as could be expected based on the filtering effect of conformity criteria as observed in the basic example. Furthermore, it is seen that both effects positively contribute towards a lower fraction defectives.

4 Influence on structural reliability calculations

Since some batches of concrete are rejected after conformity assessment, the mean of the strength distribution of accepted concrete lots increases and the standard deviation decreases compared to the population submitted for conformity inspection [Rackwitz, 1983], [Caspeele, 2010]. The probability that the compressive concrete strength is lower than a specified design value, will thus be reduced. As a consequence, this influences the failure probability (or the more conventional reliability index) of concrete structures.

As an example, the influence of the EN 206-1 conformity criteria on the reliability index is investigated, considering prior distributions based on an extensive database of strength results [Rackwitz, 1983]. The filter effect with respect to the concrete strength distribution is calculated according to the method described in [Caspeele and Taerwe, 2013] and the results for different concrete classes are provided in Table 1 in terms of the characteristics of the joint density function of the mean and standard deviation of the concrete strength distribution and in Table 2 in terms of the characteristics of the predictive concrete strength

Table 1: Characteristics (mean of the mean, standard deviation of the mean, mean of the standard deviation and standard deviation of the standard deviation) of the prior (incoming, 'i') and posterior (outgoing, 'o') joint density function for concrete strength

	C15	C25	C35	C45
$\mu_{\mu,i}$ [MPa]	28.9	37.5	46.3	52.8
$\sigma_{\mu,i}$ [MPa]	4.60	5.10	4.50	3.65
$\mu_{\sigma,i}$ [MPa]	5.02	5.45	5.01	4.42
$\sigma_{\sigma,i}$ [MPa]	1.53	1.58	1.53	1.40
$\mu_{\mu,o}$ [MPa]	30.0	39.0	47.6	54.3
$\sigma_{\mu,o}$ [MPa]	3.49	3.55	3.23	2.39
$\mu_{\sigma,o}$ [MPa]	4.79	5.13	4.69	4.00
$\sigma_{\sigma,o}$ [MPa]	1.37	1.41	1.32	1.11

Table 2: Mean and standard deviation of prior and posterior predictive concrete strength distributions as well as the corresponding fraction defectives

	C15	C25	C35	C45
μ_i [MPa]	28.9	37.5	46.3	52.8
σ_i [MPa]	6.80	7.44	6.73	5.74
θ_i [%]	2.9	5.0	4.7	8.1
μ_o [MPa]	30.0	39.2	47.8	54.3
σ_o [MPa]	5.90	6.13	5.61	4.64
θ_o [%]	0.4	0.8	1.0	1.9
μ_o / μ_i [-]	1.035	1.046	1.031	1.027
σ_o / σ_i [-]	0.867	0.825	0.834	0.808

distribution. These values are based on (1) a lognormal distribution for the concrete strength, (2) prior information according to [Rackwitz, 1983] and (3) conformity control according to EN206-1.

Consequently, the structural reliability of reinforced concrete members are analysed by a full-probabilistic Level II analysis (i.e. the First-Order Reliability Method, FORM). In general, the limit state function is written as follows:

$$g(X) = K_R\, R - K_E\,(G + Q_{50}) \tag{15}$$

where

K_R	is the model uncertainty related to the structural resistance R
K_E	is the model uncertainty related to the load effect E
G	is the permanent load
Q_{50}	is the imposed load related here to a 50-year reference period.

The structural resistance of a short axially loaded concrete column under compression is considered to be given by:

$$R = R(\underline{X}) = K_{R,col}\,(\alpha_{cc}\, b\, h\, f_c + \rho\, b\, h\, f_y) \tag{16}$$

while for a concrete beam subjected to bending this is given by:

$$R = R(\underline{X}) = K_{R,bend}\, \rho\, b\,(h-a)\, f_y \left[h - a - 0.5\,\rho\,(h-a)\,\frac{f_y}{\alpha_{cc}\, f_c} \right] \tag{17}$$

where

$K_{R,col}$	is the resistance uncertainty for a column subjected to compression
$K_{R,bend}$	is the resistance uncertainty for a beam subjected to bending
α_{cc}	is the coefficient for the long-term effects on f_c
b	is the width of the cross-section of the concrete column/beam
h	is the height of the cross-section of the concrete column/beam
f_c	is the concrete compressive strength
ρ	is the reinforcement ratio

f_y is the yield strength of the reinforcement

a is the distance between the axis of the reinforcement and the surface of the concrete beam.

Further, the design guidelines provided in EN 1992-1-1 : 2008 are taken into account when defining the design values of the variables and for the determination of the design loads. The concrete strength class considered in the design is C30/37. The probabilistic models for the basic variables as explained in [Caspeele et al., 2013] are also considered here. These probabilistic models are in accordance with the Probabilistic Model Code [JCSS, 2006] and other background materials of JCSS [Holicky and Sykora, 2011].

Based on FORM calculations and taking into account the filter effect of the conformity criteria on the predictive strength distributions according to the results of Table 2 (i.e. considering different possible concrete classes which are actually casted), the results of the filter effect with respect to the structural reliability index are given in Table 3 for the case of concrete columns subjected to compression and in Table 4 for the case of concrete beams subjected to bending. Different reinforcement ratios ρ are considered, while the load ratio $\chi = Q_k/(G_k + Q_k)$ based on the characteristic values is kept constant with a value of 0.5.

Table 3: Influence of conformity control by EN 206-1 on the reliability index of a concrete column, considering different concrete classes and reinforcement ratios

		C15	C25	C35	C45
	β_i [-]	3.11	3.81	4.53	5.01
ρ = 2%	β_o [-]	3.23	4.08	4.72	5.17
	β_o / β_i [-]	1.07	1.07	1.04	1.03
	β_i [-]	3.28	3.82	4.38	4.76
ρ = 4%	β_o [-]	3.43	4.02	4.52	4.88
	β_o / β_i [-]	1.05	1.05	1.03	1.03
	β_i [-]	3.36	3.80	4.26	4.58
ρ = 6%	β_o [-]	3.47	3.95	4.37	4.68
	β_o / β_i [-]	1.03	1.04	1.03	1.02

The influence of conformity control on the structural reliability index is more pronounced for the example of the concrete column compared to the example of the concrete beam. Furthermore, in case of the concrete column the filter effect decreases with an increasing reinforcement ratio, while in case of the concrete beam it increases with increasing reinforcement ratio ρ. The increase in reliability index is about 2% to 7% in case of the column. The filtering effect on the reliability index will increase even more in case the incoming population is of lower quality compared to the general concrete quality considered as prior distributions in [Rackwitz, 1983].

5 Variance reducing effect of concrete conformity control on structural reliability analyses under parameter uncertainties

As the standard deviation of the concrete strength distribution decreases, conformity criteria also have a filtering effect on the uncertainty with respect to the reliability index, i.e. they have a variance reducing effect when the reliability index itself is considered as a variable due to the parameter uncertainties of the influencing variable in the structural reliability analysis. As a result, the structural reliability index becomes less vulnerable with respect to parameter uncertainties. In this section, a methodology to quantify this effect is explained and consequently the effect is quantified for concrete columns and beams in case of concrete strength conformity control according to EN 206-1:2000.

Table 4: Influence of conformity control by EN 206-1 on the reliability index of a concrete slab, considering different concrete classes and reinforcement ratios

		C15	C25	C35	C45
	β_i [-]	3.57	3.62	3.66	3.67
ρ = 0.5%	β_o [-]	3.58	3.63	3.66	3.67
	β_o / β_i [-]	1.00	1.00	1.00	1.00
	β_i [-]	3.58	3.70	3.77	3.80
ρ = 1.0%	β_o [-]	3.61	3.72	3.78	3.81
	β_o / β_i [-]	1.01	1.01	1.00	1.00
	β_i [-]	3.57	3.78	3.89	3.95
ρ = 1.5%	β_o [-]	3.63	3.81	3.91	3.95
	β_o / β_i [-]	1.02	1.01	1.00	1.00

The first-order Taylor approximation with respect to the standard deviation of the uncertain reliability index $B \equiv \beta(\underline{\vartheta})$ as mentioned in [Der Kiureghian, 2008] can be used for estimating the variance of the uncertain reliability index considering parameter uncertainties. In case only the parameter uncertainties with respect to the concrete strength distribution are taken into account, i.e. regarding the mean μ_{fc} and standard deviation σ_{fc}, the Taylor approximation reduces to:

$$\sigma_B^2 \cong \left(\nabla_{\underline{\Theta}}\beta\right)^T_{\underline{M}_\vartheta} \underline{\Sigma}_{\vartheta\vartheta} \left(\nabla_{\underline{\Theta}}\beta\right)_{\underline{M}_\vartheta}$$

$$= \begin{bmatrix} \left.\dfrac{\partial\beta}{\partial\mu_{f_c}}\right|_{\mu_{\mu_{f_c}},\mu_{\sigma_{f_c}}} & \left.\dfrac{\partial\beta}{\partial\sigma_{f_c}}\right|_{\mu_{\mu_{f_c}},\mu_{\sigma_{f_c}}} \end{bmatrix} \begin{bmatrix} \sigma^2_{\mu_{f_c}} & 0 \\ 0 & \sigma^2_{\sigma_{f_c}} \end{bmatrix} \begin{bmatrix} \left.\dfrac{\partial\beta}{\partial\mu_{f_c}}\right|_{\mu_{\mu_{f_c}},\mu_{\sigma_{f_c}}} \\ \left.\dfrac{\partial\beta}{\partial\sigma_{f_c}}\right|_{\mu_{\mu_{f_c}},\mu_{\sigma_{f_c}}} \end{bmatrix} \tag{18}$$

and the standard deviation of the uncertain reliability index B thus yields:

$$\sigma_B \cong \sqrt{\left(\left.\frac{\partial\beta}{\partial\mu_{f_c}}\right|_{\mu_{\mu_{f_c}},\mu_{\sigma_{f_c}}}\right)^2 \sigma^2_{\mu_{f_c}} + \left(\left.\frac{\partial\beta}{\partial\sigma_{f_c}}\right|_{\mu_{\mu_{f_c}},\mu_{\sigma_{f_c}}}\right)^2 \sigma^2_{\sigma_{f_c}}} \tag{19}$$

In [Ditlevsen & Madsen, 1996] the following expression is derived for the derivative of the reliability index β to a certain parameter ϑ (adjusted with respect to the sign convention):

$$\frac{\partial\beta}{\partial\vartheta} = -\underline{\alpha}^T \cdot \left(\frac{\partial\underline{u}}{\partial\vartheta}\right)_{\underline{u}=\underline{u}^*} \tag{20}$$

with $\underline{\alpha}$ the vector of sensitivity factors and $\underline{u}^*$ the design point in the Gaussian space of the variables.

In case the variables of the structural reliability problem are considered independent of the concrete strength variable X and this variable is normally distributed, the derivatives of the reliability index with respect to the mean and standard deviation yield:

$$\frac{\partial \beta}{\partial \mu_{f_c}} = -\alpha_{f_c} \frac{\partial}{\partial \mu_{f_c}} \left(\frac{x - \mu_{f_c}}{\sigma_{f_c}} \right) = \frac{\alpha_{f_c}}{\sigma_{f_c}} \tag{21}$$

$$\frac{\partial \beta}{\partial \sigma_{f_c}} = -\alpha_{f_c} \frac{\partial}{\partial \sigma_{f_c}} \left(\frac{x - \mu_{f_c}}{\sigma_{f_c}} \right) = \alpha_{f_c} \left(\frac{x - \mu_{f_c}}{\sigma_{f_c}^2} \right)_{u_{f_c} = u_{f_c}^*} = \frac{\alpha_{f_c} u_{f_c}^*}{\sigma_{f_c}} = -\frac{\alpha_{f_c}^2 \beta}{\sigma_{f_c}} \tag{22}$$

with α_{fc} the sensitivity factor for the concrete strength.

Based on Equation 19 and taking into account Equations 21 and 22, the standard deviation of the uncertain reliability index B is found to be:

$$\sigma_B \cong \sqrt{\left(\frac{\alpha_{f_c}}{\mu_{\sigma_{f_c}}} \right)^2 \sigma_{\mu_{f_c}}^2 + \left(\frac{\alpha_{f_c}^2 \mu_B}{\mu_{\sigma_{f_c}}} \right)^2 \sigma_{\sigma_{f_c}}^2} \tag{23}$$

with

$$\mu_B \cong g(\underline{M}_{\vartheta}) = \beta(\mu_{\mu_{f_c}}, \mu_{\sigma_{f_c}})$$

Based on Equation 23, the filtering effect of conformity control on the standard deviation of the uncertain reliability index is given by:

$$\frac{\sigma_{B,o}}{\sigma_{B,i}} \cong \frac{\sqrt{\left(\frac{\alpha_{f_c,o}}{\mu_{\sigma_{f_c,o}}} \right)^2 \sigma_{\mu_{f_c,o}}^2 + \left(\frac{\alpha_{f_c,o}^2 \mu_{B,o}}{\mu_{\sigma_{f_c,o}}} \right)^2 \sigma_{\sigma_{f_c,o}}^2}}{\sqrt{\left(\frac{\alpha_{f_c,i}}{\mu_{\sigma_{f_c,i}}} \right)^2 \sigma_{\mu_{f_c,i}}^2 + \left(\frac{\alpha_{f_c,i}^2 \mu_{B,i}}{\mu_{\sigma_{f_c,i}}} \right)^2 \sigma_{\sigma_{f_c,i}}^2}} \tag{24}$$

which enables to quantify the variance reducing capacity of conformity control on the reliability index, considering the parameter uncertainties with respect to concrete strength distributions.

Similar as in section 4, the filter effect of the EN 206-1 criteria on the uncertain resistance reliability index B is quantified using FORM analyses. Consider first the example of a concrete column. Taking into account the characteristics of the prior and posterior joint density functions as given in Table 1, the results of the FORM analyses are provided in Table 5 for different concrete classes and different reinforcement ratios ρ. In this table also the filter effect with respect to the predictive reliability index is quantified, with the predictive reliability index β_{pred} calculated according to [Der Kiureghian, 2008]:

Table 5: Influence of conformity control by EN 206-1 on the reliability index (considered as a variable) of concrete columns, considering different concrete classes and reinforcement ratios

		C15	C25	C35	C45
	$\mu_{B,i}$ [-]	3.32	4.02	4.68	5.10
	$\mu_{B,o}$ [-]	3.45	4.16	4.78	5.21
	$\alpha_{fc,i}$ [-]	0.39	0.35	0.28	0.22
ρ = 2%	$\alpha_{fc,o}$ [-]	0.36	0.32	0.25	0.20
	$\mu_{B,o} / \mu_{B,i}$ [-]	1.039	1.035	1.021	1.022
	$\sigma_{B,o} / \sigma_{B,i}$ [-]	0.750	0.701	0.696	0.671
	$\beta_{pred,o} / \beta_{pred,i}$ [-]	1.070	1.066	1.040	1.032
	$\mu_{B,i}$ [-]	3.42	3.96	4.48	4.82
	$\mu_{B,o}$ [-]	3.51	4.07	4.56	4.91
	$\alpha_{fc,i}$ [-]	0.30	0.28	0.23	0.19
ρ = 4%	$\alpha_{fc,o}$ [-]	0.28	0.26	0.21	0.17
	$\mu_{B,o} / \mu_{B,i}$ [-]	1.026	1.028	1.018	1.019
	$\sigma_{B,o} / \sigma_{B,i}$ [-]	0.753	0.704	0.709	0.655
	$\beta_{pred,o} / \beta_{pred,i}$ [-]	1.044	1.047	1.030	1.027
	$\mu_{B,i}$ [-]	3.46	3.90	4.33	4.62
	$\mu_{B,o}$ [-]	3.53	3.99	4.40	4.70
	$\alpha_{fc,i}$ [-]	0.25	0.24	0.20	0.16
ρ = 6%	$\alpha_{fc,o}$ [-]	0.23	0.22	0.18	0.15
	$\mu_{B,o} / \mu_{B,i}$ [-]	1.020	1.023	1.016	1.017
	$\sigma_{B,o} / \sigma_{B,i}$ [-]	0.738	0.689	0.696	0.687
	$\beta_{pred,o} / \beta_{pred,i}$ [-]	1.033	1.037	1.025	1.022

$$\beta_{pred} \cong \frac{\mu_B}{\sqrt{1+\sigma_B^2}} \tag{25}$$

From these results, it is observed that the variance reducing capacity of conformity control with respect to the reliability index B is considerably higher than the filter effect with respect to the mean of the reliability index μ_B. The ratio of the standard deviation of B considering conformity control to the standard deviation without considering the effect of conformity control ranges from around 0.65 to 0.75 for the examples under consideration. This filter effect will be even more pronounced in case the incoming quality is lower than in the suggested prior information which was used for the analyses.
These results also indicate that in case conformity control is taken into account, the sensitivity factor α_{fc} only changes slightly compared to the structural reliability analysis without taking into account conformity control. Similar results for the case of concrete beams can be observed in Table 6. Although the filter effect with respect to the mean of the uncertain reliability index and the predictive reliability index is significantly lower than for the case of concrete columns, the filter effect with respect to the standard deviation is of a comparable magnitude and ranges from around 0.50 to 0.75.

6 Conclusions

- In general, quality control has a favourable effect on material properties due to the fact that the existence of quality requirements (such as conformity criteria) compels producers to deliver high quality products in order to avoid rejection by quality assessment. This effect has a beneficial influence on the probabilistic modelling of material properties of accepted inspection lots and also influences structural reliability analyses.
- As conformity criteria are used to reject or accept concrete lots, they pose a filtering effect with respect to the predictive distribution of the material property under consideration. As a result, the quality (in terms of fraction defectives) of accepted inspection lots will be higher than the quality of the incoming population which is submitted for conformity control or compared to the situation where no conformity assessment takes place. In case of a one-sided conformity criterion for concrete strength for example, the mean of the posterior predictive strength distribution of the material property will increase, while the standard deviation will decrease compared to the prior predictive strength distribution.
- A Bayesian updating methodology applied to update prior distributions, using numerical integration and taking into account OC-curves calculated by numerical Monte Carlo

Table 6: Influence of conformity control by EN 206-1 on the reliability index (considered as a variable) of concrete beams, considering different concrete classes and reinforcement ratios

		C15	C25	C35	C45
	$\mu_{B,i}$ [-]	3.58	3.63	3.66	3.67
	$\mu_{B,o}$ [-]	3.59	3.63	3.66	3.67
	$\alpha_{fc,i}$ [-]	0.04	0.02	0.01	0.01
ρ = 0.5%	$\alpha_{fc,o}$ [-]	0.03	0.02	0.01	0.01
	$\mu_{B,o} / \mu_{B,i}$ [-]	1.003	1.000	1.000	1.000
	$\sigma_{B,o} / \sigma_{B,i}$ [-]	0.596	0.740	0.767	0.724
	$\beta_{pred,o} / \beta_{pred,i}$ [-]	1.003	1.000	1.000	1.000
	$\mu_{B,i}$ [-]	3.60	3.71	3.77	3.80
	$\mu_{B,o}$ [-]	3.62	3.72	3.78	3.81
	$\alpha_{fc,i}$ [-]	0.08	0.05	0.03	0.02
ρ = 1.0%	$\alpha_{fc,o}$ [-]	0.07	0.04	0.02	0.02
	$\mu_{B,o} / \mu_{B,i}$ [-]	1.006	1.003	1.003	1.003
	$\sigma_{B,o} / \sigma_{B,i}$ [-]	0.696	0.592	0.511	0.724
	$\beta_{pred,o} / \beta_{pred,i}$ [-]	1.007	1.003	1.003	1.003
	$\mu_{B,i}$ [-]	3.61	3.79	3.90	3.95
	$\mu_{B,o}$ [-]	3.65	3.82	3.91	3.96
	$\alpha_{fc,i}$ [-]	0.14	0.08	0.04	0.03
ρ = 1.5%	$\alpha_{fc,o}$ [-]	0.12	0.07	0.04	0.02
	$\mu_{B,o} / \mu_{B,i}$ [-]	1.011	1.008	1.003	1.003
	$\sigma_{B,o} / \sigma_{B,i}$ [-]	0.682	0.648	0.767	0.482
	$\beta_{pred,o} / \beta_{pred,i}$ [-]	1.016	1.010	1.003	1.003

simulations. This general methodology enables to consider complex conformity criteria as well as to take into account autocorrelation between consecutive test results.

- The effect of conformity control on the structural reliability index of concrete structures can be quantified using a First-Order Reliability Method (FORM) in function of different incoming qualities as well as for different reinforcement ratios. It was found that:
  - The filtering effect of the EN 206-1 conformity criteria on the resistance reliability index is much more pronounced for the case of concrete columns compared to the case of concrete beams. The increase in reliability index is about 2% to 7% in case of concrete columns.
  - In case of concrete columns, the filter effect decreases with an increasing reinforcement ratio ρ, while for concrete beams the filter effect increases with increasing reinforcement ratio ρ.
- Further, also the variance reducing capacity of conformity control on the reliability index was quantified. The variance reducing effect of the EN 206-1 conformity criteria was analysed based on FORM analyses for concrete columns and beams. The major conclusions drawn from this investigation, are:
  - The variance reducing capacity of conformity control with respect to the reliability index B (considered as a variable) is considerably higher than the filter effect with respect to the mean value of the reliability index.
  - The ratio of the standard deviation of B taking into account conformity control to the standard deviation when conformity control is not taken into account, is around 60% to 75% for the examples under consideration.
  - The results indicated that the sensitivity factor α_{fc} only changes slightly when conformity control is or is not considered.
- The filter effects of conformity criteria will be even more pronounced in case the incoming population is of a lower quality than the general quality of normal ready-mixed concrete production (considered here as prior information).

Acknowledgment

I would like to thank Prof. Ton Vrouwenvelder (TU Delft/TNO) for the fruitful discussions and suggestions received during my PhD defence, afterwards during my postdoctoral research period and still in my current professorship position at Ghent University. Especially the discussions related to the use of Bayesian techniques, the background philosophy of the Eurocodes and especially on partial (safety) factors have been very

helpful and stimulating. I look forward to continue some unsolved discussions during the years to come, maybe also with respect to the use of the term *simplified level II approach*, for which it seems we both do not want to yield our opinion.

Figure 8: Picture taken after PhD defence R. Caspeele (2010; part of the examination committee). From left to right: R. Boel, L. Taerwe, T. Vrouwenvelder, R. Caspeele, G. De Schutter, N. De Belie

Literature

Box, G., Tiao, G., *Bayesian Inference in Statistical Analysis*, Addison-Wesly Publishing Company, USA, 1973.

Caspeele, R., *Probabilistic evaluation of conformity control and the use of Bayesian updating techniques in the framework of safety analyses of concrete structures*, PhD thesis, Ghent University, Ghent, Belgium, 2010.

Caspeele, R., Sykora, M. and Taerwe, L. (2013) Influence of quality control on structural reliability: assessment using a Bayesian approach. *Materials and Structures*, available as online first (DOI 10.1617/s11527-013-0048-y) (in press)

Caspeele, R. and Taerwe, L. (2011a). Statistical comparison of data from concrete families in ready-mixed concrete plants. *Structural Concrete* 12: 148-154.

Caspeele, R. and Taerwe, L. (2011b). Probabilistic evaluation of conformity criteria for concrete families. *Materials and Structures* 44: 1219-1231.

Caspeele, R. and Taerwe, L. (2013). Numerical Bayesian updating of prior distributions for concrete strength properties considering conformity control. *Advances in Concrete Construction* 1(1): 85-102.

Der Kiureghian, A. (2008). Analysis of structural reliability under parameter uncertainties. *Probabilistic Engineering Mechanics* 23: 351-358.

Diamantidis, D. et al., *Probabilistic Assessment of Existing Structures*, RILEM Publications S.A.R.L., France, 2001.

Ditlevsen, O. and Madsen, H.O. *Structural Reliability Methods*, John Wiley & Sons, UK, 1996.

Gelman, A., Carlin, J.B., Stern, H.S. and Rubin, D.B., *Bayesian Data Analysis*, second ed., Chapman & Hall/CRC, USA, 2004.

Holický, M. and Sýkora, M. (2011). Conventional probabilistic models for calibration of codes. In: Faber, M.H., Köhler, J., Nishijima, K. (Eds.), *Proc. ICASP11 11th International Conference on Applications of Statistics and Probability in Civil Engineering*, 1-4 August 2011, ETH Zurich, Switzerland: 969-976.

Jacinto, L., Pipa, M., Neves, L.A.C. and Santos, L.O. (2012). Probabilistic models for mechanical properties of prestressing strands. *Construction and Building Materials* 36: 84-89.

JCSS, *Probabilistic Model Code*, Internet publication, http://www.jcss.ethz.ch; 2006

Rackwitz, R. (1979). Über die Wirkung von Abnahmekontrollen auf das Verteilungsgesetz von normalen Produktionsprozessen bei bekannter Standardabweichung (in German). *Materialprüfung* 21: 122-124.

Rackwitz, R. (1983). Predictive distribution of strength under control. *Materials and Structures* 16: 259-267.

Raiffa, H. and Schlaifer, R., *Applied Statistical Decision Theory*, MIT Press, Cambridge, Massachusetts Institute of Technology, USA, 1969.

Taerwe, L., *Aspects of the stochastic nature of concrete strength including compliance control* (in Dutch), PhD thesis, Ghent University, Ghent, Belgium, 1985.

Taerwe, L. (1987). Influence of autocorrelation on OC-lines of compliance criteria for concrete strength. *Materials and Structures* 20: 418-427.

Taerwe, L. (1988). Evaluation of compound compliance criteria for concrete strength. *Materials and Structures* 21: 13-20.

Taerwe, L. and Caspeele, R. (2006). Conformity control of concrete: some basic aspects. In: Proske, D., Mehdianpour, M. and Gucma, L. (Eds.), *Proc. 4th International Probabilistic Symposium*, Berlin: 57-70.

Wisniewsky, D.F., Cruz, P.J.S., Henriques, A.A.R. and Simões, R.A.D. (2012). Probabilistic models for mechanical properties of concrete, reinforcing steel and pre-stressing steel. *Structure and Infrastructure Engineering* 8(2): 111-123.