A numerical study on interacting damage mechanisms in FRP laminated composite plates

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Experimental testing and numerical analysis often travel in tandem in a design process. Recent advances in novel numerical techniques to simulate mesh objective crack propagation offers a great potential for an accurate yet efficient damage analysis of composite laminates. This paper presents a numerical study on computational modelling of mesh independent matrix cracking and delamination in laminated composite plates subjected to out-of-plane loading. The analyses are performed with emphasis on a better understanding of the damage development and the interaction between different damage mechanisms. In particular, the effect of fibre orientation, the interface material properties and splitting cracks on damage development and overall response of the laminate is studied.

Key words: Composite laminated plates, mesh objective matrix cracking, delamination, impact

1 Introduction

Safe and reliable design of composite structures relies on accurate predictions of strength and damage tolerance. Development of reliable computational tools, for virtual testing of composite laminates, has been a subject of interest for physicists, material and design engineers for many decades. Computational failure models, on one hand, allow to numerically predict the strength and damage tolerance of a laminate. On the other hand, they are helpful in understanding the damage development, the mechanisms involved in progressive failure and the interaction between different damage mechanisms in composite laminates under various geometries, material properties and loading conditions. With an advent of advanced numerical tools such as mesh-independent discrete fracture models (e.g. extended finite element method (XFEM) [Moes, Dolbow and Belytschko, 1999; Wells and Sluys 2001], phantom node method [Mergheim and Steinmann, 2006]) have opened the gate to efficient numerical simulation of fracture phenomena. Impact-induced damage in composites may cause significant damage in the form of matrix cracking and delamination [Choi, Downs, and Chang, 1991]. These damage mechanisms are generally confined within thin bands of high straining, called localization zones. Computational models based on interface elements [Allix and Ladeveze, 1992; Liu, 1994; Hashagen, Schellekens, de Borst, and Parisch, 1995; Collombet, Bonni, and Lataillade, 1996; de Moura and Goncalves, 2004; Bouvet, Castanie, Bizeul, and Barrau, 2009] allow efficient simulation of fracture phenomena compared to continuum damage, plasticity and failure based models [Choi and Chang, 1992; Luo, Green, and Morrison, 1999; Zhao and Cho, 2004; Iannucci and Ankersen, 2006; Donadon, Iannucci, Falzon, Hodgkinson, and de Almeida, 2008]. However, the use of interface elements require the crack to propagate along predefined locations and the finite element mesh is also required to be aligned with the crack geometry. Thus, it requires different finite element meshes to be generated for different ply orientations with special attention to element stacking in thickness direction, e.g. a ±45 laminate requires a finite element mesh of diamond shape elements [Bouvet, Castanie, Bizeul, and Barrau, 2009]. XFEM or phantom node methods, on the other hand, allow mesh objective modelling of fracture propagation. Consequently, the same finite element mesh can be used for different stacking sequences. Moreover, crack locations and directions need not to be specified in advance when using XFEM or phantom node methods. Such an approach has been utilized to model the in-plane response of laminated composites in [Iarve, 2003; van der Meer and Sluys, 2009] and out-of-plane response of laminated composite plates in [Ahmed and Sluys, 2013].

In the present paper, damage development in laminated composite plates is studied. In particular, the aim of the present study is to investigate the effects of different factors affecting the response and damage growth in a laminated plate subjected to out-of-plane quasi-static loading.



Figure 1: Mesoscopic failure model for laminated plates/shells; (a) laminated shell and solid-like shell element, (b) Progressive failure model



Figure 2: Discontinuous solid-like shell element; (a) Discontinuity in the shell mid-surface, (b) Discontinuity in the shell director, (c) Discontinuity in the internal stretching field due to unsymmetric bending on both sides of crack. Zoom-in shows shift of material centre line (initially coincident with geometric centre line) to the side which is stretched.

2 Discontinuous progressive failure model

Finite element simulations described in this paper are based on a meso-scopic progressive failure model of Ahmed and Sluys [2013]. Each ply of the laminate is modelled with a single layer of solid-like shell elements in thickness direction, figure 1. The solid-like shell element resembles an eight-node solid element. In addition to geometrical nodes, the

element contains four independent internal nodes at the corners of the element midsurface. Due to the three-dimensional nature of a solid-like shell element, a complete threedimensional stress state is obtained. This is crucial for modelling delamination phenomena. The element has only displacement degrees of freedom which avoids the need of a complicated update of rotational degrees of freedom in geometrical nonlinear problems, as is the case with classical shell finite elements.

To model mesh independent matrix cracking/splitting through a finite element mesh of solid-like shell elements, a discontinuous solid-like shell (DSLS) model [Ahmed, van der Meer, and Sluys, 2012] is used. A phantom node method [Mergheim and Steinmann, 2006] is used to incorporate the discontinuity in the shell mid-surface, shell director and in the internal stretching field, figure 2. This enables the element to model arbitrarily propagating cracks through a finite element mesh. Fracture is modelled as a gradual process using a cohesive zone model. The crack surfaces are assumed to be normal to the shell mid-surface. The crack growth direction is taken equal to the fibre direction.

The process of delamination cracking is modelled using the shell interface model. The numerical framework is able to capture the interaction between matrix cracking and delamination in laminated composites subjected to transverse loading. The discontinuity introduced by matrix cracking in one or both planes of the delamination interface is properly modelled, i.e. if the delamination interface contains a crack, a discontinuity is also introduced in the shell interface element. If a discontinuity in the shell interface element is not taken into account, it may result in incorrect prediction of the load carrying capacity [Ahmed and Sluys, 2013].

The bulk material is modelled with orthotropic material properties. In order to solve the non-linear system of equations, the Newton Raphson iterative scheme is used.

3 Numerical examples

In this section several numerical examples are presented to show the performance of the model and to study damage development in fibre-reinforced laminated composite plates. Firstly, a cross-ply laminated plate is analyzed and the numerical results are validated against experimental observations. Next, a series of numerical analyses are presented to



Figure 3: A GFRP laminated plate under transverse loading

Ply level properties	
Longitudinal Young's modus, E_{11} (GPa)	37.9
Transverse Young's modulus, $E_{22} = E_{33}$ (GPa)	9.07
In-plane shear modulus, $G_{12} = G_{12}$ (GPa)	3.72
Poisson's ratio, $v_{12}=v_{13}$	0.3573
Poisson's ratio, v_{23}	0.4
Mode I fracture toughness, G _{Ic} (N/mm)	0.2
Mode II fracture toughness, <i>G</i> _{IIc} (N/mm)	0.6
Transverse tensile strength, f_{2t} (MPa)	20
In-plane shear strength, f_{12} (MPa)	35.5

Table 1: Material properties used for GFRP laminated plate

study and understand damage evolution in FRP laminated plates and to highlight the effects of different factors such as fibre orientation, interface material properties and matrix splitting on damage evolution. The interaction between different damage mechanisms in out-of-plane loaded composite laminates is studied.

3.1 Verification of a square GFRP laminated plate

A square, [0₁₀/90₂₀/0₁₀] graphite-fiber reinforced laminated plate is analyzed. Geometry and boundary conditions of the plate are shown in figure 3. The plate is simply supported on all edges and is loaded with a central transverse load. Material properties used for the analysis are given in table 1. Figure 4 shows the load displacement response in comparison with the experimental results of Kamiya, Sekiine and Yagishita [1998]. The numerical results show good agreement with the experimental results. Different labels on the graph shows sequence of different damage mechanisms. It is evident that formation of matrix



Figure 4: Load-displacement curve of a GFRP laminated plate; A – initiation of cohesive matrix cracking in ply-1, B – initiation of cohesive matrix cracking in ply-2, C – initiation of a traction-free matrix crack in ply-1, D – initiation of delamination at interface-2, E – initiation of a traction-free matrix crack in ply-2, F – initiation of delamination at interface-1.

cracking triggers delamination damage. Figure 5 shows delamination damage at the interfaces and matrix cracking in the plies. A peanut shape delamination damage area under mode-II fracture is evident from figure 5b. More details on damage evolution in FRP laminated plates, sequence of different damage mechanisms and reasons of their initiation and growth are discussed in the subsequent sub-sections.

3.2 Characteristic damage mechanisms

Matrix cracking was observed to be an initial damage mode of impact damage in laminated plates [Choi, Downs and Chang, 1991; Choi and Chang, 1992]. Matrix cracking can be classified into two types. Firstly, a bending matrix crack may be generated at the surface ply of the laminate, see figure 6a. This type of cracking initiates delamination along the interface of the cracked ply in a peanut shape, oriented along the fibre direction of the cracked ply. Secondly, a shear matrix crack in the inner plies of the laminate may appear, figure 6b. This type of cracking generates a substantial delamination at the bottom interface of the cracked ply, stretched along the fibre orientation of the lower ply and a small confined delamination at the top interface of the cracked ply.





Figure 5: Delamination damage at the interfaces and matrix cracking in the connecting plies at P = 2000 N; (a) Delamination at interface-2 and matrix cracking in ply-3 (horizontal cracks) and ply-2 (vertical cracks), (b) Delamination at interface-1 and matrix cracking in ply-1 (horizontal cracks) and ply-2 (vertical cracks). Dark lines indicate traction-free portion of the cracks. Colour figures are available at the HERON website.



Figure 6: Impact induced characteristic damage mechanisms of FRP laminated plates; (a) Delamination due to surface bending cracks, (b) Delamination due to inner shear cracks



0° Fiber direction

Figure 7: Four-ply, unsymmetric laminated plate

Ply level properties	MAT1	MAT2
Longitudinal Young's modus, E_{11} (GPa)	140	120
Transverse Young's modulus, $E_{22} = E_{33}$ (GPa)	10	10.5
In-plane shear modulus, $G_{12} = G_{12}$ (GPa)	5	3.48
Poisson's ratio, $v_{12}=v_{13}$	0.21	0.3
Poisson's ratio, v_{23}	0.21	0.5
Mode I fracture toughness, G _{Ic} (N/mm)	0.3	0.26
Mode II fracture toughness, G_{IIc} (N/mm)	0.7	1.002
Transverse tensile strength, f_{2t} (MPa)	50	30
In-plane shear strength, f_{12} (MPa)	30	60

Table 2: Material properties used for CFRP laminated plate analyses

[Choi, Down and Chang, 1991; Choi and Chang, 1992] and briefly explained above, in laminated plates subjected to transverse loads. Additionally, it is also one of the aims of this example to understand the development of different damage mechanisms and their mutual interactions in laminated plates. The stacking sequence (0/90/0/90) for the present 68



Figure 8: Energy dissipation during matrix cracking and delamination damage in a four-ply laminated plate; (a) Energy dissipation due to matrix cracking, (b) Energy dissipation due to delamination.

analysis is chosen, such that both types of characteristic damage mechanisms, i.e. bending and shear crack induced mechanisms can be observed in a single laminate analysis. The geometry and boundary conditions of the laminate are shown in figure 7. The plate has a length, L = 20 mm, a width, W = 9.6 mm and a ply thickness of 0.2 mm. The analysis is performed with a laminate made of carbon-fibre-reinforced epoxy composite (CFRP). The laminate is analyzed with material MAT2. The material properties are given in table 2 and are obtained from Asp, Sjogrem and Greenhalgh [2001]. Matrix cracks are allowed to initiate and propagate with a minimum crack spacing of 0.5 mm. Due to geometrical and material symmetry, only one-half of the plate is modelled. Each ply of the laminate is modelled with a single solid-like shell element in thickness direction. A fine mesh with an average element size of 0.15 mm is used near the centre of the plate, where the damage is most likely to grow.

Figure 8 shows the energy dissipation during matrix cracking and delamination damage. Figure 9 shows the damage development over half of the plate, for two load levels. It is evident from figure 8a that matrix cracking initiates at a lower load level, $P = 30 N_{e}$ compared to delamination damage, see figure 8b. This confirms the experimental observations of [Choi, Down and Chang, 1991; Choi and Chang, 1992], that damage initiates in the matrix material of the ply. Moreover, damage initiates in the bottom ply, i.e. ply4. As the load increases, the middle crack starts to open up and grows under bending stresses along with the formation of new matrix cracks in ply4. The thick line over a crack represents the traction free portion of the crack. Propagation and formation of matrix cracking, consequently result in a progressive delamination which starts at the three interfaces at load level, P = 75N. Further increase in load causes the bending crack to propagate further and consequently causes a larger area to delaminate at the bottom (0/90)interface. The delamination grows in the shape of a half peanut, oriented along the fibre direction of the cracked ply, ply4 (figure 9b). A peanut-shape delamination zone in a laminate subjected to a point node impact has been observed experimentally by Choi, Down and Chang [1991]. The increased load also results in the formation of matrix cracking in the inner ply, ply2. It can be observed from figure 9b, that two of the middle cracks become traction free under combined bending and shearing stresses and propagates further. The formation and growth of these cracks consequently result in substantial growth of delamination at the lower interface (90/0) and small delamination at the upper interface (0/90) of the cracked ply2. These results are consistent with the experimental observations of [Choi, Down and Chang, 1991; Choi and Chang, 1992]. The numerical model fully captures the characteristic damage mechanisms of impact damage. Moreover, the damage patterns associated with bending and shear matrix cracks are also captured.



Figure 9: Damage development at two different load steps in a four-ply laminated plate; (a) Load, P = 135 N, (b) Load, P = 259.2 N. Colour figures available at the HERON website.

Another interesting feature of the damage development is the non-uniform distribution of matrix cracking within the plies. Since, the bottom ply4, has larger bending stresses and smaller shear stresses compared to the inner plies, matrix cracking is more concentrated towards the middle of the ply, figure 9. Whereas, for the case of the inner ply2, matrix cracking is more distributed and has more cracks near the edges of the plate, extended over the full width of the plate, figure 9b. This is due to high shear stresses at mid-depth of the plate. Accordingly, the delamination growth at the inner interfaces takes place under pure fracture mode II, with little or no contribution from fracture mode I. On the contrary, delamination growth at the lowermost interface takes place under mixed mode, see figure



Figure 10: Fracture mode ratio of delamination damage at different interfaces

10. Moreover, delamination damage at the inner interface (90/0), not only has a larger cohesive zone compared to delamination at the outermost interface (figure 9) but the rate of delamination growth of the inner interface (90/0) is also higher than the delamination growth rate of the outermost interface (0/90), figure 8b.

3.3 Effect of fibre orientation

In this example, the effect of fibre orientation of the connecting plies or stacking sequence, on the damage development in composite laminates, is investigated. To study the effect of stacking sequence, two un-symmetric, two-ply, square composite plates made of material MAT1 are analysed, see table 2. The material properties MAT1 are extracted from Yang and Cox [2005]. The geometry and boundary conditions of the laminated plate are given in figure 11a. The plate is clamped at all four sides and loaded quasi-statically in the middle. Due to the fact, that the discontinuous progressive failure model allows mesh-independent modelling of matrix cracking, the same finite element mesh is used for both analyses cases. Although geometrical symmetry exists for both stacking sequences, material symmetry is not present for the [0/75] laminate and the full plate needs to be modelled in the finite element analysis. The matrix cracking is allowed to initiate and propagate in the lower ply. A minimum crack spacing of 0.3mm is used for the current analyses.



Figure 11:Two-ply model geometry and analysis results; (a) Model geometry and boundary conditions, (b) Load-displacement curves

Figure 12 shows the matrix cracking in the lower ply and interface delamination for both stacking sequences. A dashed thick line over middle cracks represents the traction free portion of the crack. It can be observed from the figure, that delamination damage grows in a peanut-shape, oriented along the fibre direction of the lower ply. The numerical results are consistent with the experimental observations of Liu [1988].

It can also be observed from figure 12, that the area of delamination for the [0/90] laminate is more extended compared to the [0/75] laminate. This is due to a mismatch of bending stiffness of the connecting plies to an interface, Liu [1988]. As the difference in fibre orientation of the connecting plies increases, the area of delamination also increases.



Figure 12: Matrix cracking in the lower ply and delamination damage at the interface for (0/90) and (0/75) laminates - Load, P = 101 N; (a) [0/90] laminate, (b) [0/75] laminate. Colour figures available at the HERON website.

Hence, a uni-directional laminate [0/0] will have no delamination compared to a cross-ply [0/90] laminate. However, it is noted, that the area of delamination also increases with the



Figure 13: Energy dissipation due to matrix cracking and delamination damage in a two-ply laminate with stacking sequence [0/90] and [0/75]; (a) Energy dissipation due to matrix cracking, (b) Energy dissipation due to delamination.

increase in material an-isotropy, Ahmed and Sluys [2013]. Figures 13a and 13b compare the energy dissipation due to matrix cracking and delamination damage for the two stacking sequences. It is noted, that in the present model the damage is not a single parameter. Instead damage is classified based on different damage modes (e.g. matrix cracking and delamination). For this reason, comparison is made at similar damage level, for the two cases (i.e. [0/90] and [0/75]), for each damage mode individually. Load capacity at similar

matrix cracking is obtained from figure 13a. Similarly, load capacity at similar delamination damage is obtained from figure 13b. It is confirmed from these plots, that a cross-ply laminate dissipates more energy, both in matrix cracking and delamination damage, compared to a [0/75] laminate. As a result of this the [0/75] laminate shows a stiffer behavior and larger load carrying capacity compared to a cross-ply laminate, see figure 11b.

In contrast to delamination damage, matrix cracking appears to be less influenced by the stacking sequence, figure 13. Moreover, the number of matrix cracks, the area over which matrix cracking is smeared and the length of matrix cracks are not significantly different for both stacking sequences, see figure 12.

3.4 Effect of splitting cracks

Here, we explore the effect of matrix splitting on the response and fracture characteristics of laminated plates. A laminate subjected to out-of-plane loading may develop splitting cracks due to axial stresses in addition to distributed matrix cracking due to bending and/or shear stresses. A four-ply [0/90/0/90], un-symmetric laminate is analyzed. The model geometry and boundary conditions are the same as used in section 3.2. The material properties used for the analysis correspond to MAT1, see table 2. Two analysis cases are studied. In the first case, named model-A, matrix cracks are allowed to initiate and propagate in all four plies, i.e. bending and shear dominated matrix cracking in plies 2 and 4, and splitting cracks in plies 1 and 3. In the second analysis case, named model-B, the cracks are only allowed to initiate and propagate in plies 2 and 4, whereas no cracks are allowed to grow in plies 1 and 3.

Figure 14a compares the load-displacement curves of the two models. It can be observed, that the global responses of both models are the same. Figure 14b compares the energy dissipated during delamination damage at different interfaces for the two models. It can be observed, that the dissipated energy for all interfaces is almost the same for the two models.

Figure 15 compares the energy dissipated during matrix cracking and splitting, for the two models, in different plies. Firstly, it can be observed from figure 15a, that the energy dissipated by the splitting cracks is smaller than the energy dissipated during matrix

cracking. This suggests that the assumption of ignoring matrix splitting in the rest of the examples of this paper is valid. However, this conclusion cannot be generalized, due to the fact that this depends upon the geometry of the laminate, the loading and boundary conditions and material properties. In general, the presence of splitting cracks, in addition to bending/shear cracks, may influence the laminate response and damage development. This will be further discussed hereafter.

Comparing the matrix cracking energy dissipation of the two models (figure 15), it can be observed that the presence of splitting cracks affects the energy dissipation due to bending/shear dominated matrix cracking, in plies 2 and 4. For the case of ply2, which is in compression, model-A shows less energy dissipation compared to model-B. Whereas, for the case of ply4, which is in tension, the energy dissipated in model-A is larger compared to model-B. Thus, it can be concluded that the presence of splitting cracks does affect the initiation and growth of bending/shear cracks. Moreover, since these cracks act as stress enhancers, which triggers delamination damage, inclusion or exclusion of splitting cracks, in numerical modelling of progressive failure in laminated composites under impact, may affect the damage development and the overall laminate response.

Figure 16 shows the delamination damage at the interfaces and matrix cracking of the connecting plies. It is evident from the figures, that the two models show almost similar delamination areas. Moreover, the number of matrix cracks and the area over which they are smeared out are approximately the same for both models.

3.5 Effect of interface material properties

In order to investigate the effect of interface material properties on damage mechanisms, a two-ply [0/90] laminated plate is analyzed. The geometry and boundary conditions of the plate a re the same as used in section 3.2. However, instead of four plies, the laminate analyzed in this section consists of two plies. The plate is analyzed with materials MAT1 and MAT2, see table 2. Note, that the material sets MAT1 and MAT2 are only slightly



Figure 14: Four-ply laminated plate analysis results; (a) Load-displacement curves, (b) Energy dissipation due to delamination

different with respect to their bulk material properties, however, their interface material properties are significantly different from each other. The material set MAT1 has higher normal and weaker shear strengths compared to MAT2. This implies that MAT1 interfacial



Figure 15: Energy dissipation due to matrix cracking in different plies of four-ply laminated plate

behaviour is more brittle in a normal opening mode and more ductile in a shear opening mode compared to MAT2. The analyses are performed with a minimum crack spacing of 0.5mm. The delamination damage at the interface and matrix cracking in the lower ply of the laminate are shown in figure 17, for both analysis cases, corresponding to different load



Figure 16: Damage development at different interfaces of four-ply laminated plate at load, P = 100 N; Left: model-A, Right: model-B

levels. It is observed, that the number of cohesive matrix cracks is higher for the case of MAT2 compared to MAT1. Moreover, most of the cohesive cracks for the case of MAT2, extended to the full width of the specimen and are smeared over a larger area along the length of the plate compared to MAT1. It can also be observed, that the length of a traction free crack is longer in MAT1 compared to MAT2. Accordingly, the length of the interaction



Figure 17: Damage development in two-ply laminated plate at load levels P = 30.0, 48.0 and 73.0 N; Left: MAT1, Right: MAT2

between the two damage mechanisms. Moreover, the width of the delaminated area is also larger in MAT1, while for the case of MAT2, the shape of the delaminated area is elongated.

Similar observations can be made from figure 18, in which MAT1 shows limited energy dissipation during matrix cracking and larger energy dissipation during delamination, compared to MAT2. Moreover, it can be observed, that for the case of MAT2, matrix cracking initiates at lower load level compared to MAT1. On the contrary, delamination damage initiates with a slight delay for MAT2 compared to MAT1. Difference in delamination area corresponding to a particular amount of matrix cracking can be obtained from figure 18. For example, at dissipated energy through matrix cracking of



Figure 18: Energy dissipation in a two-ply laminated plate with different material properties; (a) Delamination energy dissipation, (b) Matrix cracking energy dissipation

0.2 N/mm, the load capacity, for the case of MAT1 and MAT2, is approximately 55 N and 30 N, respectively. The dissipated energy through delamination damage at the corresponding load levels is 0.7 N/mm and 0.05 N/mm, respectively. This result indicates, that at a particular matrix crack length, MAT1 experiences more delamination damage compared to MAT2.

It can be concluded from the above analyses, that two materials with almost similar bulk material properties but different interface material properties may show significantly different damage mechanisms which may affect the overall response of the laminate.

4 Conclusions

A numerical study on damage development in composite laminated plates subjected to out-of-plane loading has been presented. The study helped in understanding the mechanics of damage development, the sequence of different damage mechanisms, the distribution of energy dissipation through different mechanisms and their mutual interaction in composite plates. Additionally, the effects of fibre orientation of the connecting plies, the interface material properties and the presence of splitting cracks on damage development and overall response of laminated composite plates have been studied. The discontinuous progressive failure model was able to simulate and capture the characteristic damage modes associated with bending and shear matrix cracking.

It is observed, that the first damage event in a composite laminated plate subjected to transverse load, is matrix cracking in the outermost ply, on the tension side under the loading point. As the matrix crack opens or becomes traction free it triggers delamination damage. Therefore, matrix cracks are one of the primary sources of the initiation of delamination damage. Moreover, the energy dissipated during matrix cracking is smaller than the energy dissipated during delamination.

The area of delamination at an interface depends upon the fibre orientation of the connecting plies. If the difference in fibre orientation of the connecting plies is large, the area of delamination will also be large. However, the same is not true for the amount of matrix cracking. The matrix cracking damage is only mildly affected by the stacking sequence compared to delamination damage. This suggests that the presence of matrix cracks is not the only cause of delamination, instead matrix cracking and fibre orientation of the connecting plies both determine the size of delaminated area at an interface.

Damage growth is also dependent upon the interface material properties. Laminates with similar bulk material properties but different interface material properties may show different damage mechanisms. Therefore, the interface properties also may influence the overall damage tolerance and strength of a laminate. Moreover, the presence of splitting

cracks may also influence the energy dissipation mechanisms of a laminate. Therefore, for accurate predictions of laminate strength and damage tolerance, it is necessary to also include splitting cracks in numerical analyses.

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