Structural design for ponding of rainwater on roof structures

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Ponding of rainwater is a special load case that can lead to roof collapse. In Dutch building practice the most frequently occurring damage cases are failures of flat roof structures caused by ponding of rainwater. In the Dutch code for loadings and deformations NEN6702 [1] and the Dutch guidelines for practice regarding ponding NPR 6703 [2], principles and guidelines for the determination of rainwater loads are given. The Dutch code [1] prescribes a complex iterative procedure for ponding of rainwater. Today, there are a number of computer software programs available to support the structural designer in this iteration method. However, to keep insight in the process of rainwater ponding, a simple design method for ponding of slightly sloping flat (steel) roof structures was developed. The method is described in the first part of this article. In the second part a sensitivity analysis for design and construction inaccuracies is presented. It is shown that roofs, that are seemingly stiff enough to withstand ponding, need partial safety factors substantially greater than normally used to account for construction inaccuracies. A proposal for the partial safety factor related to roof stiffness and construction inaccuracies is given.

Key words: Ponding, rainwater, roof, collapse, construction, sensitivity analysis, safety, design, calculation, structural behaviour.

1 Introduction

Rainwater ponding occurs by deformation of flat roofs caused by rainwater. Due to the deformation, extra rainwater flows to the lower area of the roof, resulting in a larger loading with a larger deformation, resulting in more rainwater flowing towards this area, etc. In case of well-designed and constructed flat roofs, the deformation will reach a limit state, with an equilibrium, whereby the roof structure has enough capacity to bear the rainwater loading. In other cases, when flat roofs are not well designed and constructed,

the deformation process continues without limit as long as water is being added, leading to a failure of the roof. Rainwater ponding can be prevented by adequate construction measures. The rainwater load is of minor importance compared with other live loads on the roof, like snow and wind loads, in case the roof structure has a sufficient slope, stiffness and number of emergency drains. What combination of slope, stiffness and number of emergency drains is 'sufficient' cannot be determined beforehand. In [1] principles for the determination of rainwater loading are given. In case the rainwater loading is known, roof structures can be assessed on their bearing capacity for rainwater loading, using material related design codes. In [1] an iterative calculation method is prescribed, based on the theory of applied mechanics, to determine the deformation of the roof structure due to rainwater ponding. In [2] guidelines for the determination of water loads for ponding of rainwater are given.
For a fundamental roof ponding problem, namely a simply supported beam resting on rigid supports that are at the same level, [1] gives a safe approach, whereby implicitly the effects of iterations have been taken into account. Also a safe alternative for an iterative procedure to determine the rainwater loading is given based on an estimation of the maximum deformation of the roof.
For a large number of common roof structures design methods, which are implicitly dealing with the iterative effects of rainwater ponding, are not available. Iterative procedures are time-consuming and complex. For that reason they are often not used, leading to failure of the roof structure as the ultimate consequence. In this article, based on [3], design methods are presented, also for complex but realistic roof structures (e.g. purlins on flexible beams), taking the iterative nature of rainwater ponding implicitly into account. Therefore, the iterative procedure no longer is necessary. Assessment of roof structures of all kind of structural materials, using the presented design methods, is simple and provides insight in the underlying mechanisms.

Recent international publications on the problem of rainwater ponding on flat and sloping roofs are scarce [4-9]. The topic is studied in Italy and The Netherlands but must be relevant to other countries as well, especially where (nearly) flat roofs are built and heavy rainfall occurs frequently.

In this article, at first the principles for the load case of rainwater ponding are treated. After that, beams on rigid supports are discussed, followed by beams on flexible supports. It has been found possible to derive a set of equations for roof structures with an orthogonal set of flexible girders supported by flexible main beams, to analyse the roof structures in a simple way in case of rainwater ponding. Then, a sensitivity analysis is carried out showing that partial safety factors have to be substantially greater than normally used to account for construction inaccuracies.

2 Principles for rainwater ponding

In [1] the principles for rainwater ponding are given as treated hereafter. Figure 1 shows schematically a cross section over the roof edge. The emergency drain opening, with width b and height h, is situated at a distance h_{nd} over the roof. It is assumed that only a height of d_{nd} of the opening is used. The water level d_{hw} at the roof edge then is:

$$d_{hw} = d_{nd} + h_{nd} \tag{1}$$

Other principles for the load case rainwater ponding are as follows:

- the load is bound to a location;
- water drainage through one or more regular water drains is not possible because of obstruction;
- water drainage over the roof edge or through the opening of the emergency drain(s) is possible.

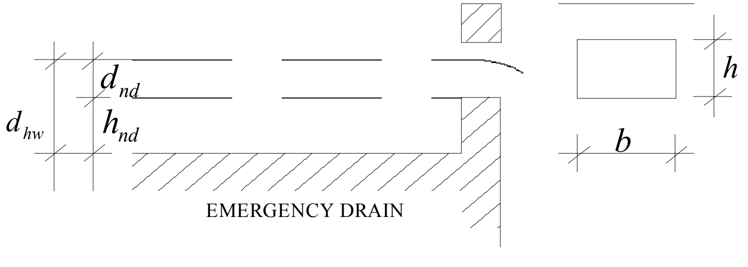


Figure 1: Roof edge with rainwater and emergency drain opening

According to [1], the rainwater load $\overline{p}$ is:

$$\overline{p} = (d_{hw} + d_n)\gamma \tag{2}$$

in which:

γ density of water (10 kN/m³);

d_n water level caused by the deformation of the roof structure by permanent load and water ponding, determined with the iterative procedure of the code [1].

Hereafter, a number of common load cases on roof beams are analytically treated without use of an iterative calculation method.

3 Beams on rigid supports

In case of beams rigidly supported at both ends, we can distinguish the following load cases (Figure 2):

A. Uniformly distributed load;

B. Triangular load;

C. Trapezoidal load;

D. Partial triangular load.

Load case A is for a horizontal roof; the load cases B to D are for sloping roofs.

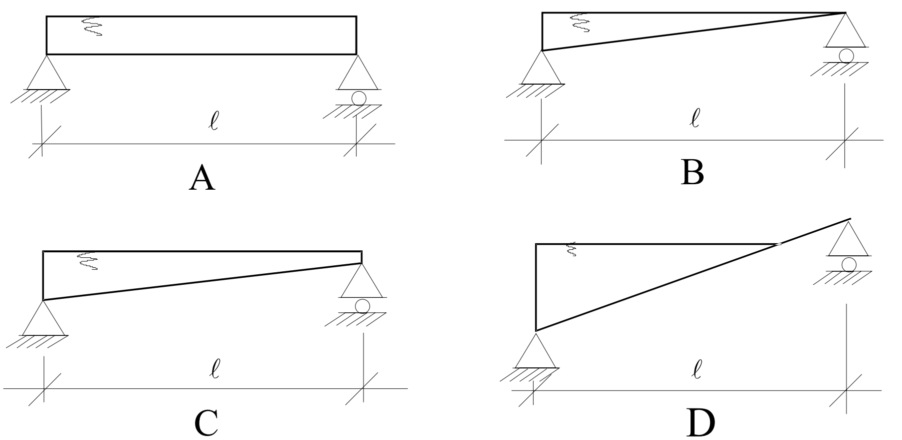


Figure 2: Load cases for rigid supported beams

The centre-to-centre distance of the beams is *a*, the bending stiffness is *EI* and the water level over the not-deformed roof at the roof edge d_{hw}.

3.1 *Uniformly distributed load (load case A)*

The first load case to be considered is a uniformly distributed load (Figure 2A). The uniformly distributed load

$q = a\gamma d_{hw}$ can be approached by a sinusoidal load with an amplitude $\hat{q} = \frac{4}{\pi} \cdot a\gamma d_{hw}$ (Figure 3). The maximum bending moment and additional deformation caused by a sinusoidal load correspond very well with the values due to a uniformly distributed load, while the mathematical relations between load, bending moment and deflection are simpler.

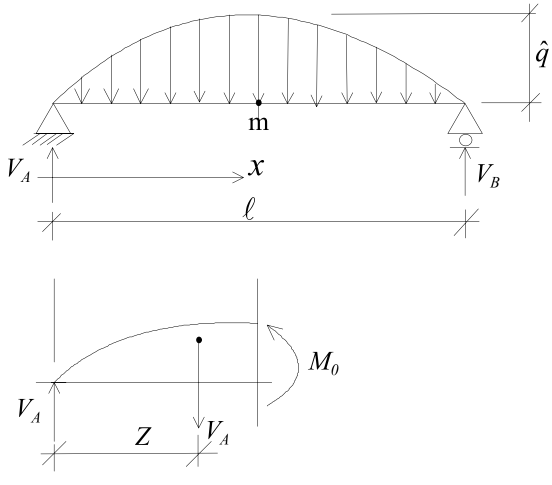


Figure 3: Sinusoidal load

Suppose:

$$\frac{4}{\pi} \cdot d_{hw} = \hat{d}_{hw} \tag{3}$$

then:

$$\hat{q} = a\,\gamma\,\hat{d}_{hw} \tag{4}$$

Then, for a sinusoidal load the following expression holds:

$$q_x = \hat{q} \sin \frac{\pi x}{\ell} \tag{5}$$

The support reactions are then as follows:

$$V_A = V_B = \int_0^{\frac{\ell}{2}} \hat{q} \sin \frac{\pi x}{\ell} dx = \frac{\ell}{\pi} \cdot \hat{q} \tag{6}$$

and the first-order bending moment in the middle is (Figure 3):

$$M_0 = V_A \cdot Z \tag{7}$$

It can be derived [3] that $Z = \ell / \pi$, so that the bending moment in the middle is:

$$M_0 = V_A \cdot Z = \frac{\ell^2}{\pi^2} \hat{q} \tag{8}$$

This bending moment M_0 corresponds very well with the bending moment as a result of a uniformly distributed load q, since:

$$M_0 = \frac{\ell^2}{\pi^2} \hat{q} = \frac{\ell^2}{\pi^2} \cdot \frac{4}{\pi} q = \frac{q\ell^2}{7.752} \approx \frac{q\ell^2}{8} \tag{9}$$

The first-order deflection in the middle of the beam $\hat{\delta}_0$ can be determined by considering the moment diagram area as load (Figure 4). The bending moment caused by this load, divided by the bending stiffness EI, results in $\hat{\delta}_0$.

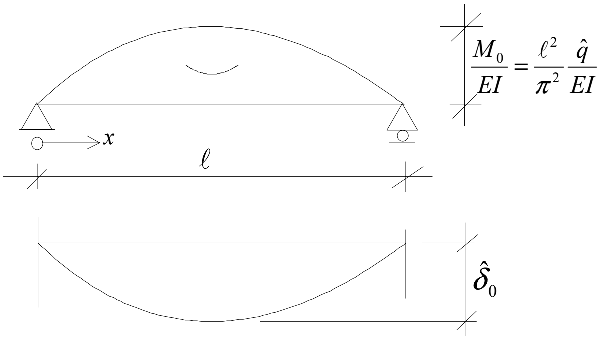


Figure 4: Moment diagram area as sinusoidal load

In this way the following expression is obtained:

$$\hat{\delta}_0 = \frac{\ell^2}{\pi^2} \left(\frac{\ell^2}{\pi^2} \cdot \frac{\hat{q}}{EI} \right) = \frac{\ell^4}{\pi^4} \frac{\hat{q}}{EI} \tag{10}$$

Also, this value corresponds very well with the deflection caused by a uniformly distributed load q, since:

$$\hat{\delta}_0 = \frac{\ell^4}{\pi^4} \frac{\hat{q}}{EI} = \frac{\ell^4 \cdot 4}{\pi^4 \; \pi} \cdot \frac{q}{EI} = \frac{5.02 q\ell^4}{384 EI} \approx \frac{5 q\ell^4}{384 EI} \tag{11}$$

As a result of this first-order deflection $\hat{\delta}_0$ there will be a water flow till the original water level is reached, giving an additional deflection $\hat{\delta}_1$ caused by a corresponding load $\hat{q}_0 = a \gamma \hat{\delta}_0$.

The additional deflection $\hat{\delta}_1$ can be calculated analogous to eqn. (10):

$$\hat{\delta}_1 = \frac{\ell^4 \hat{q}_0}{\pi^4 EI} = \frac{\ell^4 \cdot a\gamma\hat{\delta}_0}{\pi^4 EI} = \frac{\ell^4 a\gamma}{\pi^4 EI} \frac{\ell^4}{\pi^4} \frac{a\gamma\hat{d}_{hw}}{EI} \tag{12}$$

As a result of $\hat{\delta}_1$ there will be an additional deflection $\hat{\delta}_2$, etc.

The total deflection due to the water load $\hat{\delta}_{end}$ is the sum $\hat{\delta}_{end} = \hat{\delta}_0 + \hat{\delta}_1 + \hat{\delta}_2 ....$

or:

$$\hat{\delta}_{end} = \left(\frac{\ell^4}{\pi^4}\frac{a\gamma\hat{d}_{hw}}{EI}\right) \times \left(1 + \frac{\ell^4 a\gamma}{\pi^4 EI} + \left(\frac{\ell^4 a\gamma}{\pi^4 EI}\right)^2 + ...\right) \tag{13}$$

If $\frac{\ell^4 a\gamma}{\pi^4 EI} = 1$, than $\hat{\delta}_{end}$ is just unlimited.

The corresponding value of the bending stiffness $EI = \frac{a\gamma\ell^4}{\pi^4}$ is defined as the critical bending stiffness EI_{cr}:

$$EI_{cr} = \frac{a\gamma\ell^4}{\pi^4} \tag{14}$$

Suppose:

$$\frac{EI}{EI_{cr}} = n \tag{15}$$

then eqn. (13) can be rewritten as:

$$\hat{\delta}_{end} = \hat{\delta}_0\left(1 + \frac{1}{n} + \frac{1}{n^2}..........\right) \tag{16}$$

Now, if:

$$\frac{1}{n} < 1 \tag{17}$$

then:

$$\hat{\delta}_{end} = \frac{\hat{\delta}_0}{1 - \frac{1}{n}} = \hat{\delta}_0 \cdot \frac{n}{n-1} \tag{18}$$

With:

$$\hat{\delta}_0 = \frac{\ell^4}{\pi^4}\frac{a\gamma\hat{d}_{hw}}{EI} = \frac{EI_{cr}}{EI} \cdot \hat{d}_{hw} = \frac{\hat{d}_{hw}}{n} \tag{19}$$

it is found that:

$$\hat{\delta}_{end} = \hat{\delta}_0 \frac{n}{n-1} = \frac{\hat{d}_{hw}}{n-1} \tag{20}$$

The deflection in the middle of the beam can simply be calculated using $\hat{d}_{hw}$ and n.

A small value of n gives large deflections (and bending moments) and has to be avoided.

The water load $\hat{q}_{end}$ as a result of $\hat{\delta}_{end}$ is $a\,\gamma\,\hat{\delta}_{end}$.

The increase of M_0 is:

$$\Delta M = \frac{\ell^2}{\pi^2}\hat{q}_{end} = \frac{\ell^2}{\pi^2} a\,\gamma\,\hat{\delta}_{end} \tag{21}$$

so that:

$$M_{end} = M_0 + \Delta M \tag{22}$$

3.2 *Triangular load (load case B)*

The next load case to be considered is a triangular load (Figure 2B). This load occurs in case of a sloping roof. The results for load case B can be derived directly from those for load case A. The deflection in the middle is (based on symmetry considerations) exactly half of the deflection caused by a uniformly distributed load (load case A):

$$\hat{\delta}_0 = \frac{\hat{d}_{hw}}{2n} \tag{23}$$

and:

$$\hat{\delta}_{end} = \frac{\hat{d}_{hw}}{2n}\cdot\frac{n}{n-1} = \frac{\hat{d}_{hw}}{2(n-1)} \tag{24}$$

3.3 *Trapezoidal load (load case C)*

The third load case to be considered is a trapezoidal load (Figures 2C and 5). Also this load case can be derived from the previous cases, considering a trapezoidal load as the sum of a uniformly distributed load and a triangular load [3], resulting in:

$$\hat{\delta}_{end} = \left(\frac{\hat{d}_{hw1}}{n} + \frac{\hat{d}_{hw2}}{2n}\right)\frac{n}{n-1} \tag{25}$$

The first-order bending moment in the middle of the beam is then:

$$M_0 = \frac{\ell^2}{\pi^2}\left(\hat{d}_{hw1} + \frac{\hat{d}_{hw2}}{2}\right)a\,\gamma \tag{26}$$

and the increase of the bending moment in the middle of the beam, respectively the maximum bending moment in the beam, can again be determined with Eqns. (21) and (22) respectively.

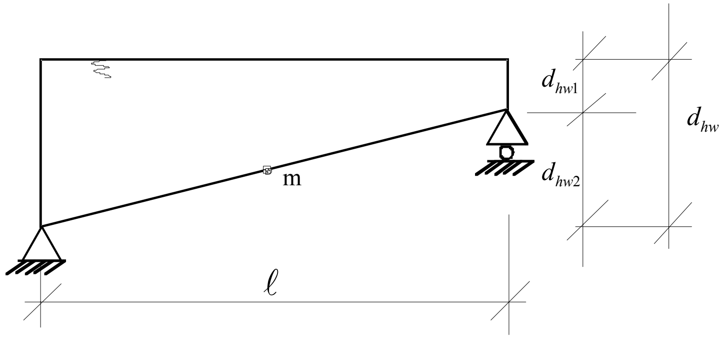


Figure 5: Trapezoidal load

3.4 ***Partial triangular load (load case D)***

In case a sloping roof is partly loaded by water, see Figures 2D and 6, a partial triangular load has to be considered over the distance *p.l* with p <1. It is hardly possible to determine second-order deflections by hand calculation so a numerical method is used.

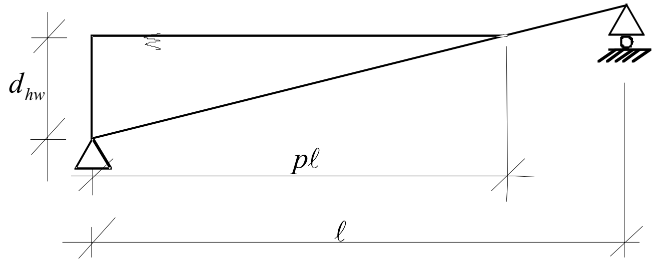


Figure 6: Partial triangular load

The numerical calculations are made with a Finite Element Program by calculating the deformations, determining the corresponding water load, again calculating the new deformations, etc. This iterative procedure is finished at the moment the increase of the deformation is smaller than 1%, as prescribed in [1]. The model used, is shown in Figure 7. Thus, the bending moments and deflections are obtained for a number of discrete values of p and n [3]. Using these moments and deflections, the coefficients C_m and C_u for bending moment and deflection respectively can be calculated with the following expressions:

$$C_{m;M_0} = \frac{M_0}{a\gamma d_{hw}\ell^2} \tag{27}$$

$$C_{m;M_{end}} = \frac{M_{end}}{a\gamma d_{hw}\ell^2} \tag{28}$$

$$C_{m;\Delta M} = \frac{\Delta M}{a\gamma d_{hw}\ell^2} = \frac{M_{end} - M_0}{a\gamma d_{hw}\ell^2} = C_{m;M_{end}} - C_{m;M_0} \tag{29}$$

$$C_{u;\hat{\delta}_0} = \frac{\hat{\delta}_0}{d_{hw}} \tag{30}$$

$$C_{u;\hat{\delta}_{end}} = \frac{\hat{\delta}_{end}}{d_{hw}} \tag{31}$$

These coefficients are listed in Table 1.

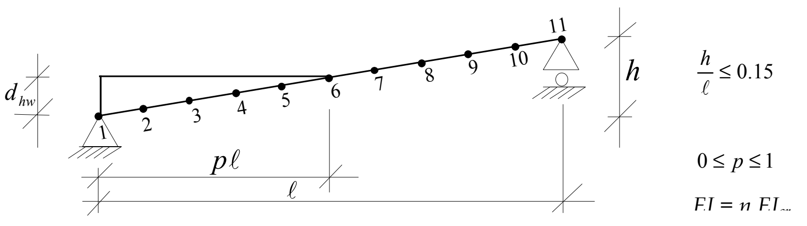


Figure 7: Model for the numerical calculation of bending moments and deflections caused by a partly triangular load

In Figure 8 the relationship is given between the moment coefficients C_m and p, for several values of n. In Figure 9 this is done for the deflection coefficients C_u.

From these figures the following can be concluded:

- If the bending stiffness increases (greater values of n), the end moment and deflection decrease.
- For $1.0 \leq n < 1.5$ the moment M_{end} and the deflection $\hat{\delta}_{end}$ are relatively great. Therefore it is recommended to design roof structures for $n \geq 1.5$.
- An increase in the length of the triangular load (a larger value of p) results in an increase of the end moment and deflection.

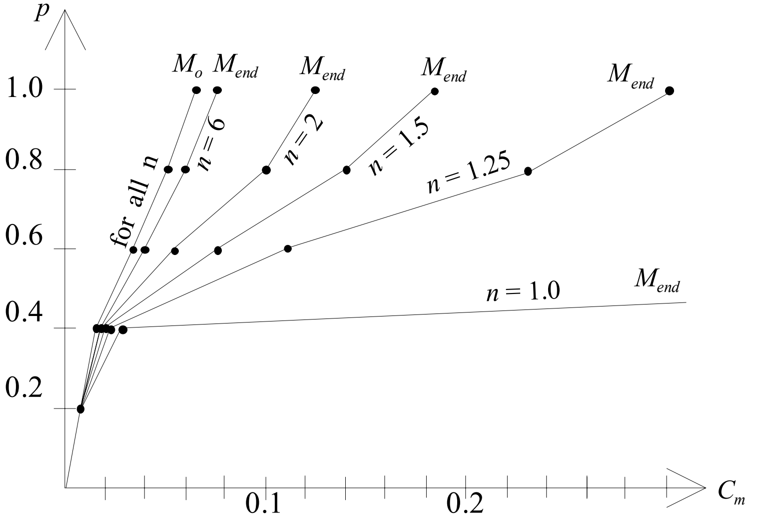


Figure 8: Moment coefficients $C_{m;M_0}$ and $C_{m;M_{end}}$ for varying values of n and p

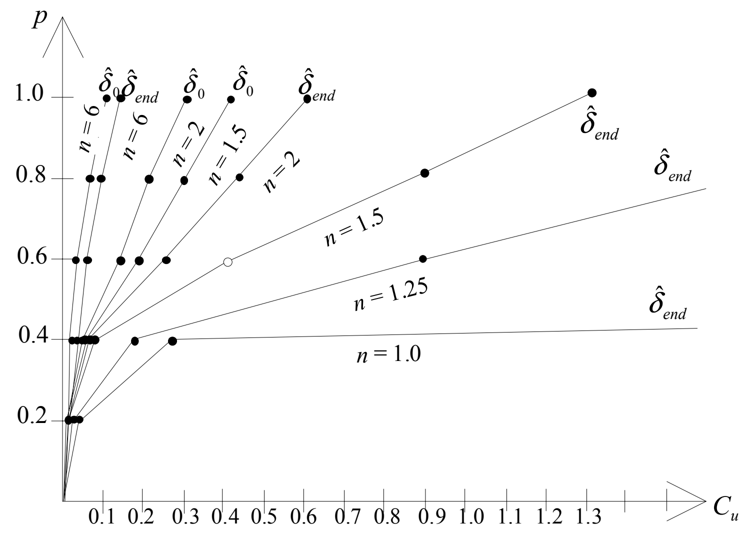


Figure 9: Deflection coefficients $C_{u;\hat{\delta}_0}$ and $C_{u;\hat{\delta}_{end}}$ at varying values of n and p

Table 1: Numerically determined coefficients C_m and C_u for different values of n and p

p		n = 1.0	n = 1.25	n = 1.5	n = 2	n = 4	n = 6	n = 8	n = 10
	$C_{u;\hat{\delta}_0}$	-	0.5039	0.4200	0.3150	0.1573	0.1049	0.0786	0.0629
	$C_{u;\hat{\delta}_{end}}$	-	2.4009	1.2637	0.6203	0.2089	0.1258	0.0897	0.0700
1.0	$C_{m;M_0}$	0.0640	0.0640	0.0640	0.0640	0.0640	0.0640	0.0640	0.0640
	$C_{m;\Delta M}$	-	0.2404	0.1246	0.0610	0.0202	0.0123	0.0087	0.0068
	$C_{m;M_{end}}$	-	0.3044	0.1886	0.1250	0.0842	0.0763	0.0727	0.0708
	$C_{u;\hat{\delta}_0}$	-	0.4116	0.3200	0.2400	0.1200	0.0800	0.0600	0.0480
	$C_{u;\hat{\delta}_{end}}$	-	1.8094	0.9304	0.4673	0.1589	0.0945	0.0678	0.0529
0.8	$C_{m;M_0}$	0.0505	0.0505	0.0505	0.0505	0.0505	0.0505	0.0505	0.0505
	$C_{m;\Delta M}$	-	0.1789	0.0906	0.0445	0.0153	0.0087	0.0063	0.0049
	$C_{m;M_{end}}$	-	0.2294	0.1411	0.0950	0.0658	0.0592	0.0568	0.0554
	$C_{u;\hat{\delta}_0}$	0.3053	0.2442	0.2035	0.1527	0.0763	0.0509	0.0382	0.0305
	$C_{u;\hat{\delta}_{end}}$	8.4875	0.8805	0.4312	0.2526	0.0907	0.0569	0.0414	0.0326
0.6	$C_{m;M_0}$	0.0342	0.0342	0.0342	0.0342	0.0342	0.0342	0.0342	0.0342
	$C_{m;\Delta M}$	0.8312	0.0788	0.0348	0.0200	0.0057	0.0036	0.0026	0.0020
	$C_{m;M_{end}}$	0.8654	0.1130	0.0690	0.0542	0.0399	0.0378	0.0368	0.0362
	$C_{u;\hat{\delta}_0}$	0.1449	0.1159	0.0966	0.0724	0.0362	0.0242	0.0181	0.0145
	$C_{u;\hat{\delta}_{end}}$	0.2693	0.1805	0.1160	0.0829	0.0386	0.0252	0.0187	0.0149
0.4	$C_{m;M_0}$	0.0175	0.0175	0.0175	0.0175	0.0175	0.0175	0.0175	0.0175
	$C_{m;\Delta M}$	0.0139	0.0091	0.0037	0.0027	0.0012	0.0008	0.0006	0.0003
	$C_{m;M_{end}}$	0.0314	0.0266	0.0212	0.0202	0.0187	0.0183	0.0181	0.0178
	$C_{u;\hat{\delta}_0}$	0.0307	0.0246	0.0205	0.0154	0.0077	0.0051	0.0038	0.0031
	$C_{u;\hat{\delta}_{end}}$	0.0355	0.0284	0.0208	0.0156	0.0077	0.0051	0.0038	0.0031
0.2	$C_{m;M_0}$	0.0045	0.0045	0.0045	0.0045	0.0045	0.0045	0.0045	0.0045
	$C_{m;\Delta M}$	0.0004	0.0004	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
	$C_{m;M_{end}}$	0.0049	0.0049	0.0046	0.0046	0.0045	0.0045	0.0045	0.0045

3.5 *Examples for beams on rigid supports*

Two examples are presented for the calculation method for rigidly supported beams under rainwater loading.

For these examples, the following assumptions are made:

- The centre-to-centre distance of the roof beams a = 5 m.
- The beam span ℓ = 15 m.
- The load factors for dead load and live load are:

  $\gamma_{f;dl} = 1.2$ and $\gamma_{f;ll} = 1.3$ respectively.

- Loads:

  The dead load of the roof plates and insulation is 0.2 kN/m^2.

  The assumed dead load of the steel beam is 0.9 kN/m.

  Then the dead load on the beam can be calculated as

  $g_{rep} = 5 \cdot 0.2 + 0.9 = 1.9$ kN/m.

  The snow load is 0.7 kN/m^2. With roof shape factor 0.8 this results in a snow load $p_{rep;s} = 5 \cdot 0.8 \cdot 0.7 = 2.8$ kN/m on the beam.

  The water load for d_{hw} = 0.1m at the roof edge is:

  $p_{rep;w} = 5 \cdot 0.1 \cdot 10 = 5$ kN/m.

  For combined dead load and snow load, the beam load is:

  $q_{d;s} = 1.2 \cdot 1.9 + 1.3 \cdot 2.8 = 5.92$ kN/m

  For combined dead load and rainwater load, the beam load is:

  $q_{d;w} = 1.2 \cdot 1.9 + 1.3 \cdot 5 = 8.78$ kN/m

  Since $p_{rep;w} > p_{rep;s}$ the water load is decisive.

- With eqn. (14) the critical bending stiffness EI_{cr} can be calculated:

  $$EI_{cr} = \frac{a \cdot \gamma \cdot \ell^4}{\pi^4} = \frac{5 \cdot 10 \cdot 15^4}{\pi^4} = 25985 \text{ kNm}^2.$$

- Now, the section profile is determined. It is advised to limit the additional deformation u_{add} of a roof e.g. by using the following requirement:

  $u_{add} \leq 0.004\ell = 0.06$ m which is based on [1]. The additional deformation u_{add} is defined as the deformation caused by the live load on a roof, in this case the water load. Thus, the water load on the roof is kept within reasonable limits by designing a relatively stiff roof structure.

  With eqn. (20), n can be calculated to fulfil this requirement:

$u_{add} = \hat{\delta}_{end} \leq 0.004 \cdot \ell = 0.06$ m. Using eqn. (20) leads to:

$0.06 \geq \frac{4}{\pi} \cdot \frac{0.1}{n-1}$ and thus: $n \geq 3.12$.

With $n = \frac{EI}{EI_{cr}} \geq 3.12$ the required bending stiffness can be calculated:

$EI > 3.12 \cdot 25985 = 81073$ kNm^2

A steel section IPE500 with $I = 48199 \cdot 10^4$ mm^4 has sufficient bending stiffness:

$EI = 2.1 \cdot 10^5 \cdot 48199 \cdot 10^4 = 101218 \cdot 10^9$ Nmm^2 = 101218 kNm^2 > 80173 kNm^2.

- For this section IPE500 the following can be calculated:
  - the additional deformation due to water load $p_{rep;w} = 5.0$ kN/m is:

    $$u_{add} = \frac{5}{384} \cdot \frac{5.0 \cdot 15^4}{101218} = 0.033 \text{ m}$$

  - the deformation due to dead load $g_{rep} = 1.9$ kN/m can be calculated as:

    $$u_{dl} = \frac{1.9}{5.0} \cdot 0.033 = 0.012 \text{ m}$$

  - The elastic moment capacity M_u for steel grade S235 can be calculated as:

    $M_u = 1928 \cdot 10^3 \cdot 235 = 453 \cdot 10^6$ Nmm = 453 kNm, with the elastic section modulus of an IPE500 section being $W = 1928 \cdot 10^3$ mm^3.

3.5.1 *Example 1: Uniformly distributed load (load case A)*

Using the assumptions and results of the previous paragraph, the following values have been considered, for the uniformly distributed load case of Figure 10:

d_{hw} = 0.1 m;

$$n = \frac{EI}{EI_{cr}} = \frac{101218}{25985} = 3.90$$

u_{dl} = 0.012 m;

a = 5 m;

ℓ = 15 m.

Equation (3), adding the deformation due to dead load, leads to:

$$\hat{d}_{hw} = \frac{4}{\pi} \cdot d_{hw} + u_{dl} = \frac{4}{\pi} 0.1 + 0.012 = 0.139 \text{ m}$$

With Eqn. (20) it is found that:

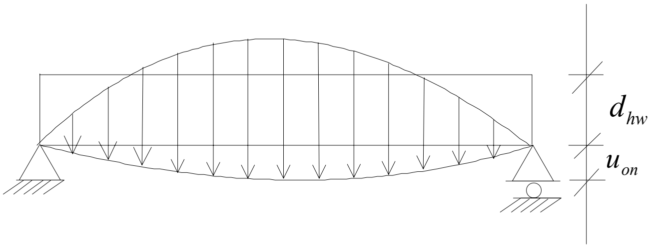


Figure 10: Example 1 – Rigidly supported beam with uniformly distributed load

$$\hat{\delta}_{end} = \frac{\hat{d}_{hw}}{n-1} = \frac{0.139}{2.90} = 0.048 \leq 0.004\ell = 0.060 \text{ m}$$

and with eqn. (21) it follows that:

$$\Delta M = \frac{\ell^2}{\pi^2} \cdot a \cdot \gamma \cdot \hat{\delta}_{end} = \frac{15^2}{\pi^2} \cdot 5 \cdot 10 \cdot 0.048 = 54.7 \text{ kNm}$$

With:

$$M_0 = \frac{1}{8} \cdot 5 \cdot 15^2 = 140.6 \text{ kNm}$$

and:

$$M_{dl} = \frac{1}{8} \cdot 1.9 \cdot 15^2 = 53.4 \text{ kNm}$$

it is found that:

$$M_d = \gamma_{f;dl} \cdot M_{dl} + \gamma_{f;ll}(M_0 + \Delta M) =$$

$$= 1.2 \cdot 53.4 + 1.3 \cdot (140.6 + 54.7) = 214.3 \text{ kNm} \leq M_u = 453 \text{ kNm}.$$

So the steel section IPE500 fulfils the strength requirement for rainwater loading. It was designed to fulfil the stiffness requirement. It can be concluded that not strength but stiffness is governing the design.

3.5.2 Example 2: Partial triangular load (load case D)

For this load case (Figure 11) again the following values have been considered:

n = 3.90;

a = 5 m;

ℓ = 15 m.

Furthermore it is assumed that:

u_{dl} = 0 m;

roof slope = 2%;

p = 0.8.

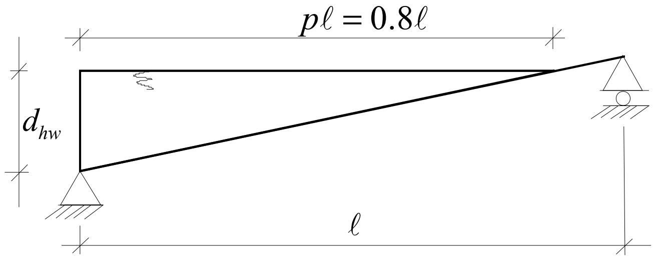


Figure 11: Example 2 – Rigidly supported beam with partial triangular load

Then:

$d_{hw} = 0.02 \cdot p\ell = 0.24\,\text{m}$

From Table 1 the values of the coefficients C_m and C_u can be determined by interpolation. For the deformations the following is obtained:

$\hat{\delta}_0 = C_{u;\hat{\delta}_0} \cdot d_{hw} = 0.1260 \cdot 0.24 = 0.0302\ \text{m}$

$\hat{\delta}_{end} = C_{u;\hat{\delta}_{end}} \cdot d_{hw} = 0.1743 \cdot 0.24 = 0.0418\ \text{m} \leq 0.004\,\ell = 0.060\ \text{m}$

The steel section IPE500 fulfils the stiffness requirement for rainwater loading.
For the bending moment the following is obtained:

$M_{end} = C_{m;M_{end}} \cdot a\,\gamma\,d_{hw}\ell^2 = 0.0673 \cdot 5 \cdot 10 \cdot 0.24 \cdot 15^2 = 181.7\ \text{kNm}$

and thus:

$M_d = \gamma_{f;dl} \cdot M_{dl} + \gamma_{f;ll} \cdot M_{end} = 1.2 \cdot 53.4 + 1.3 \cdot 181.7 = 300.3\ \text{kNm}$

So:

M_d = 300.3 kNm < M_u = 453 kNm, and the steel section IPE500 fulfils the strength requirement for rainwater loading.
The section IPE500 meets both the strength and the stiffness requirements.

4 Beams on flexible supports

Roof structures often consist of purlins on main girders. Both purlins and main girders deflect under loading on the roof. In this case, the purlins can be considered as beams flexibly supported at both ends by the main girders. For beams on flexible supports [1] prescribes an iterative calculation method to determine the rainwater loading. However, with a set of equations the total deflections of the main girder and the purlins can be calculated directly. This set of equations is derived in the next section. It is assumed that all supports are in a horizontal plane.

4.1 *Derivation of set of equations*

For the purpose of deriving the set of equations, load case A according to Figure 12 is used. From eqn. (20) it follows that:

$$n\,\hat{\delta}_{end} = \hat{d}_{hw} + \hat{\delta}_{end} \qquad (32)$$

This means that the total water level on the roof (in this case the amplitude of the replacing water level on the roof plus the deflection in the final state) is *n* times the final deflection.

The derivation of the set of equations is based on an analysis of the deflection of the main girder and the purlins. The loads are related to the deflections. Figure 13 shows the plan of the roof structure. All parameters belonging to the main girder have subscript 1; for the purlins subscript 2 is used.

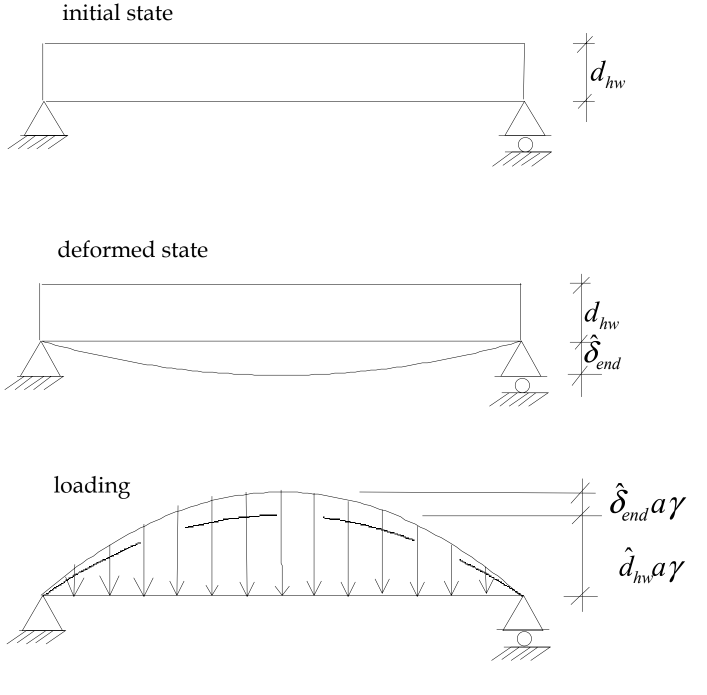


Figure 12: Load case A – deformation and load

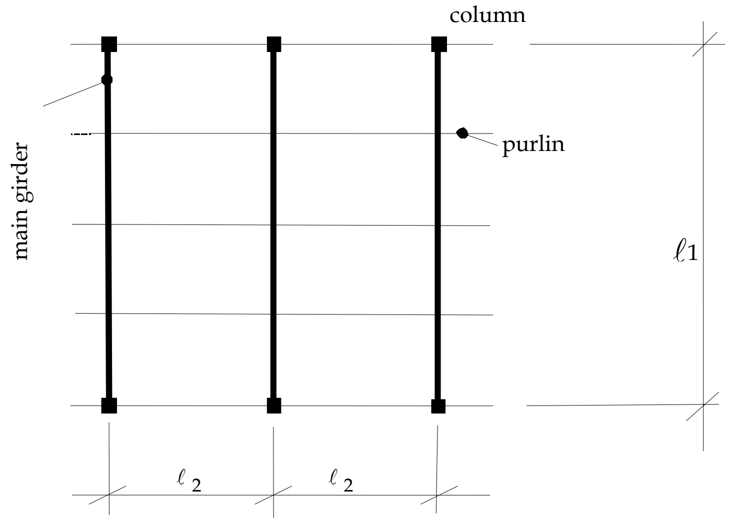


Figure 13: Ground plan of the roof structure with main girders and purlins

In case of continuous roof slabs over more supports and with sufficient height, the deflection of the roof slabs is limited. This deflection is neglected here to simplify the calculation model. Figure 14 shows an axonometric projection of the roof deflections, where subscript 1 denotes the main girders and subscript 2 denotes the purlins.

From the deflections, the water levels (and also the water loads) for the purlins and main girders can be determined. Since all loads are transformed into sinusoidal loads, the water levels will be translated into amplitudes of equivalent sinusoidal water levels.

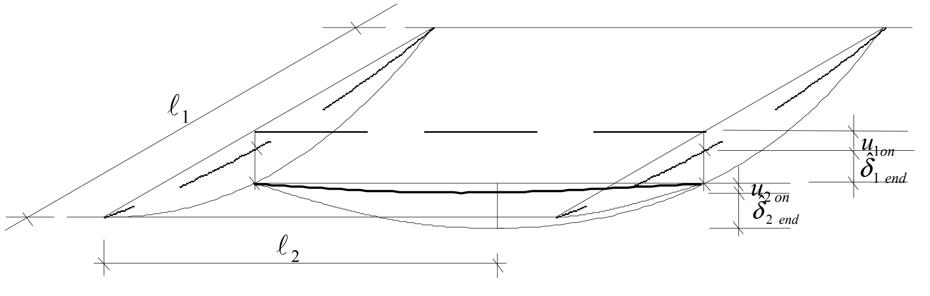
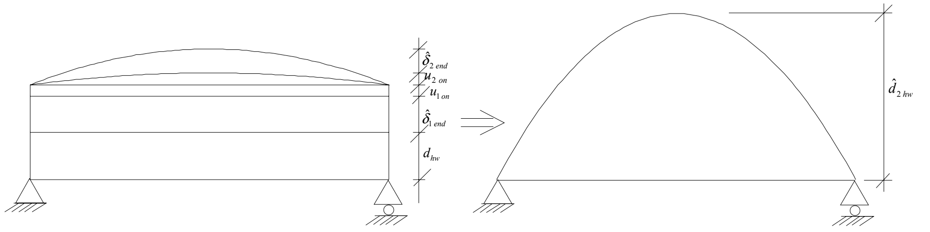


Figure 14: Deflections of the roof



a) b)

Figure 15: Water levels above the purlins and equivalent sinusoidal water level

4.1.1 Loading on purlins

The deflections of the purlins and with these also the water levels above the purlins, are shown in Figure 15a. For the maximum amplitude of the equivalent sinusoidal water level on the purlins (Figure 15b), excluding the influence of the end deflection of the purlin itself, the following expression holds:

$$\hat{d}_{2\,hw} = \frac{4}{\pi} \cdot \left(d_{hw} + \hat{\delta}_{1\,end} + u_{1\,on}\right) + u_{2\,on} \tag{33}$$

and the total load on the purlins is then:

$$\hat{q}_2 = a\gamma(\hat{d}_{2hw} + \hat{\delta}_{2end}) \tag{34}$$

In these equations, u_{1on} and u_{2on} are the deflections due to permanent loading on the main girder and the purlin respectively.

4.1.2 Loading on main girders

The deflections of the main girders and with these also the water levels above the main girders, are shown in Figure 16. Here, the contribution by the deflection of the purlins $\left(u_{2\,on} + \hat{\delta}_{2\,end}\right)$ has to be added still. The deflection of the purlins gives a water volume V under the surface $\ell_1 \cdot \ell_2$ in Figure 17. The additional water level caused by the deflection of the purlins is:

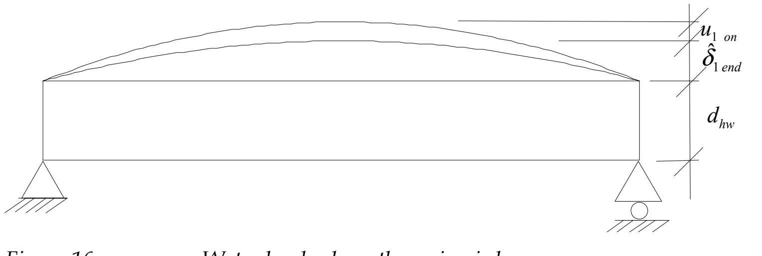


Figure 16: Water levels above the main girders

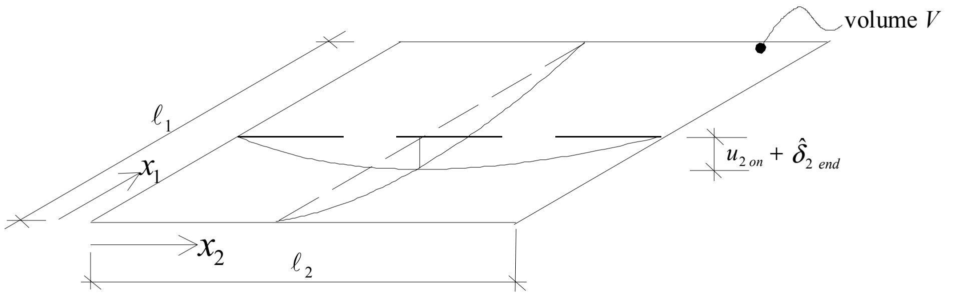


Figure 17: Contribution to the deflection of the main girders by the deflection ($u_{2on} + \hat{\delta}_{2end}$) of the purlins

$$h(x_1, x_2) = \left(u_{2\,on} + \hat{\delta}_{2\,end}\right)\sin\frac{\pi x_2}{\ell_2} \cdot \sin\frac{\pi x_1}{\ell_1} \tag{35}$$

A mean additional water level is obtained by integrating in x_2 – direction and subsequently dividing by ℓ_2. Integration yields:

$$\int_0^{\ell_2} \left(u_{2\,on} + \hat{\delta}_{2\,end}\right)\sin\frac{\pi x_2}{\ell_2} \cdot \sin\frac{\pi x_1}{\ell_1} dx_2 = \frac{2\ell_2}{\pi} \cdot \left(u_{2\,on} + \hat{\delta}_{2\,end}\right) \cdot \sin\frac{\pi x_1}{\ell_1} \tag{36}$$

and then the mean water level h_m can be calculated as:

$$h_m(x_1) = \frac{2}{\pi} \cdot \left(u_{2\,on} + \hat{\delta}_{2\,end}\right) \cdot \sin\frac{\pi x_1}{\ell_1} \tag{37}$$

So the expression for the amplitude of the mean water level is:

$$\hat{h}_m = \frac{2}{\pi} \cdot \left(u_{2on} + \hat{\delta}_{2end}\right) \approx 0.64\left(u_{2on} + \hat{\delta}_{2end}\right) \tag{38}$$

For the amplitude of the equivalent sinusoidal water level on the main girders (Figure 18), excluding the influence of the end deflection of the main girder itself, the following expression is found:

$$\hat{d}_{1hw} = \frac{4}{\pi} d_{hw} + u_{1on} + 0.64\left(u_{2on} + \hat{\delta}_{2end}\right) \tag{39}$$

and the total load on the main girders is then:

$$\hat{q}_1 = a\gamma(\hat{d}_{1hw} + \hat{\delta}_{1end}) \tag{40}$$

4.1.3 Set of equations

For the main girder, the application of eqn. (32) gives:

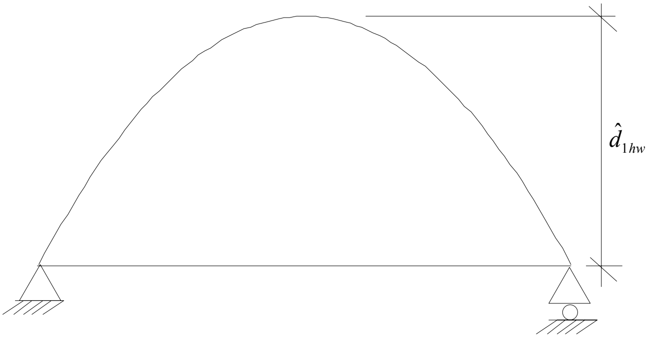


Figure 18: Equivalent sinusoidal water level on the main girders

$$n_1 \; \hat{\delta}_{1end} = \frac{4}{\pi} d_{hw} + u_{1on} + 0.64\left(u_{2on} + \hat{\delta}_{2end}\right) + \hat{\delta}_{1end} \tag{41}$$

In this case:

$$\hat{d}_{1hw} = \frac{4}{\pi} d_{hw} + u_{1on} + 0.64\left(u_{2on} + \hat{\delta}_{2end}\right) \tag{42}$$

If the bending stiffness of the purlins is infinite, so $EI_2 = \infty$, then $u_{2on} = \hat{\delta}_{2end} = 0$, and the equation for the rigidly supported beam (main girder) is obtained:

$$\hat{d}_{1hw} = \frac{4}{\pi} d_{hw} + u_{1on} \tag{43}$$

Analogously applying eqn. (32) to the purlin yields the following equation:

$$n_2 \; \hat{\delta}_{2\,end} = \frac{4}{\pi}\left(d_{hw} + \hat{\delta}_{1end} + u_{1on}\right) + u_{2on} + \hat{\delta}_{2end} \tag{44}$$

In this case:

$$\hat{d}_{2hw} = \frac{4}{\pi}\left(d_{hw} + \hat{\delta}_{1end} + u_{1on}\right) + u_{2on} \tag{45}$$

If the bending stiffness of the main girders is infinite, so $EI_1 = \infty$, then $u_{1on} = \hat{\delta}_{1end} = 0$, and the equation for the rigidly supported beam (purlin) is obtained:

$$\hat{d}_{2hw} = \frac{4}{\pi} d_{hw} + u_{2on} \tag{46}$$

The set of equations is formed by equations (41) en (44).

With these equations, $\hat{\delta}_{1\,end}$ en $\hat{\delta}_{2\,end}$ can be calculated for certain values of n_1 and n_2. Also, with this set of equations, an estimate of the values for n_1 and n_2 can be made when the maximum values of $\hat{\delta}_{1\,end}$ and $\hat{\delta}_{2\,end}$ are known. Though not required by the Dutch code [1], the limit values presented in [1] can be used to estimate maximum values of

$\hat{\delta}_{1\,end}$ and $\hat{\delta}_{2\,end}$. Thus, the water load on the roof is kept within reasonable limits by designing a relatively stiff roof structure.

The application of this set of equations is illustrated in the example in the next section for the design and calculation situation.

4.2 *Example for beams on flexible supports*

The structure as presented schematically in Figure 19 will be designed and calculated. The length of the main girders is 20 m. The centre-to-centre distance of the main girders is 10 m and the centre-to-centre distance of the purlins is 5 m. The water level at the roof edge is d_{hw}=0.15 m.

4.2.1 *Design*

First of all the set of equations is used to design the main girders and purlins. Since the values of u_{1on} and u_{2on} will be small when compared to $\hat{\delta}_{1\,end}$ and $\hat{\delta}_{2\,end}$ respectively, they are assumed to be zero. The values for n_1 en n_2 are estimated on the basis of the limiting values for $\hat{\delta}_{1\,end}$ en $\hat{\delta}_{2\,end}$ according to the code [1] by using the following criterion: $u_{add} \leq 0.004\ell$. For the main girders and the purlins this results in $\hat{\delta}_{1\,end} \leq 0.08$ m and $\hat{\delta}_{2\,end} \leq 0.04$ m respectively. Substituting the limiting values in the set of equations (41) and (44) yields:

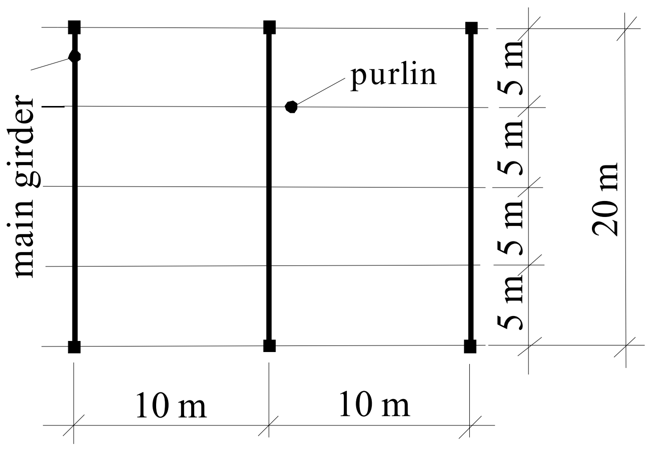


Figure 19: *Example roof structure*

- with eqn. (41): $n_1 \cdot 0.08 = \frac{4}{\pi} \cdot 0.15 + 0.64 \cdot 0.04 + 0.08 \ \rightarrow n_1 = 3.71$;
- with eqn. (44): $n_2 \cdot 0.04 = \frac{4}{\pi} \cdot (0.15 + 0.08) + 0.04 \quad \rightarrow n_2 = 8.32$.

With eqn. (14) the critical bending stiffness of the main girders can be found:

$$EI_{1cr} = \frac{a\gamma\ell^4}{\pi^4} = \frac{10 \cdot 10 \cdot 20^4}{\pi^4} = 164256 \ \text{kNm}^2.$$

For the purlins this critical bending stiffness is:

$$EI_{2cr} = \frac{a\gamma\ell^4}{\pi^4} = \frac{5 \cdot 10 \cdot 10^4}{\pi^4} = 5133 \quad \text{kNm}^2.$$

With eqn. (15) the required bending stiffness of the main girders becomes:

$$EI_1 = n_1 \ EI_{1cr} = 3.71 \cdot 164256 = 609390 \ \text{kNm}^2$$

and for the purlins:

$$EI_2 = n_2 \ EI_{2cr} = 8.32 \cdot 5133 = 42707 \ \text{kNm}^2$$

With $E = 2.1 \cdot 10^8$ kN/m² for steel, the required moment of inertia for the main girders is $I_1 = 29.0186 \cdot 10^{-4}$ m⁴ and for the purlins $I_2 = 2.0337 \cdot 10^{-4}$ m⁴.

Therefore, the following sections are chosen:

- main girders HE800A with moment of inertia $I_1 = 30.344 \cdot 10^{-4}$ m⁴, section modulus $W_1 = 7680 \cdot 10^3$ mm³ and dead load 2.24 kN/m;
- purlins IPE400 with $I_2 = 2.313 \cdot 10^{-4}$ m⁴, $W_2 = 1160 \cdot 10^3$ mm³ and dead load 0.663 kN/m.

Thus, the sections have been designed with sufficient stiffness against ponding resulting in values $n \geq 1.5$. Now, their strength has to be checked. This is done in the next section.

4.2.2 Calculation

The set of equations (41) and (44) will now be used to calculate the roof structure of Figure 19. The main girders are HE800A sections with bending stiffness $EI_1 = 2.1 \cdot 10^8 \times 30.344 \cdot 10^{-4} = 637224$ kNm² which yields:

$$n_1 = \frac{EI_1}{EI_{1cr}} = 3.88 \ .$$

The purlins are IPE400 sections with bending stiffness $EI_2 = 2.1 \cdot 10^8 \times 2.313 \cdot 10^{-4} = 48573$ kNm² which yields:

$$n_2 = \frac{EI_2}{EI_{2\,cr}} = 9.46 \;.$$

Thus, the values for n_1 and n_2 in the set of equations are known. Now the values of u_{1on} and $u_{2\,on}$ are determined. The dead load of the roof slabs including isolation and roofing is 0.2 kN/m². For the dead load on main girders and purlins respectively, the following is obtained:

$$g_{1;rep} = 10 \cdot 0.2 + 2.24 + \frac{0.663 \cdot 10}{5} = 5.566 \;\; \text{kN/m}$$

$$g_{2;rep} = 5 \cdot 0.2 + 0.663 = 1.663 \;\; \text{kN/m}$$

Then the deflections due to dead load can be calculated as:

$$u_{1on} = \frac{5}{384} \cdot \frac{5.566 \cdot 20^4}{637224} = 0.0182 \;\; \text{m}$$

$$u_{2on} = \frac{5}{384} \cdot \frac{1.663 \cdot 10^4}{48573} = 0.0045 \;\; \text{m}$$

Substituting the results in the set of equations (41) and (44) results in:

$$3.88 \;\; \hat{\delta}_{1\;end} = \frac{4}{\pi} \cdot 0.15 + 0.0182 + 0.64\left(0.0045 + \hat{\delta}_{2\;end}\right) + \hat{\delta}_{1\,end}$$

$$9.46\,\hat{\delta}_{2\;end} = \frac{4}{\pi}\left(0.15 + \hat{\delta}_{1\;end} + 0.0182\right) + 0.0045 + \hat{\delta}_{2\;end}$$

Solving this set of equations yields:

$$\hat{\delta}_{1\;end} = 0.08212 \text{ m}$$

$$\hat{\delta}_{2\;end} = 0.03821 \text{ m}$$

Now, the maximum amplitude of the equivalent sinusoidal water level on the main girders can be calculated with eqn. (41):

$$n_1\hat{\delta}_{1\;end} = \frac{4}{\pi} d_{hw} + u_{1on} + 0.64\left(u_{2on} + \hat{\delta}_{2end}\right) + \hat{\delta}_{1end} = \hat{d}_{1hw} + \hat{\delta}_{1end}$$

or

$$\hat{d}_{1hw} + \hat{\delta}_{1end} = n_1 \; \hat{\delta}_{1\;end} = 3.88 \cdot 0.08212 = 0.319 \;\; \text{m}$$

The maximum amplitude of the equivalent sinusoidal water level on the purlins can be calculated with eqn. (44):

$$\hat{d}_{2\;hw} + \hat{\delta}_{2end} = n_2 \; \hat{\delta}_{2\;end} = 9.46 \cdot 0.03821 = 0.361 \;\; \text{m}$$

The bending moment in the main girders in the ultimate limit state is then:

$$M_{1,d} = M_{1,g} \cdot \gamma_{f;dl} + M_{1,q}\,\gamma_{f;ll} \rightarrow$$

$$M_{1,d} = \frac{1}{8} \cdot g_{1;rep} \cdot \ell_1^2 \cdot \gamma_{f;dl} + \frac{\ell_1^2}{\pi^2} \cdot a\gamma(\hat{d}_{1hw} + \hat{\delta}_{1end}) \cdot \gamma_{f;ll} \rightarrow$$

$$M_{1,d} = \frac{1}{8} \cdot 5.566 \cdot 20^2 \cdot 1.2 + \frac{20^2}{\pi^2} \cdot 10 \cdot 10 \cdot 0.319 \cdot 1.3 \rightarrow$$

$$M_{1,d} = 333.9 + 1680.7 = 2014.6 \text{ kNm}$$

This bending moment results in the following elastic bending stress in the main girders:

$$\sigma_1 = \frac{2014.6 \cdot 10^6}{7680 \cdot 10^3} = 262 \text{ N/mm}^2 > \text{fyd} = 235 \text{ N/mm2}$$

The bending moment in the purlins is:

$$M_{2,d} = M_{2,g} \cdot \gamma_{f;dl} + M_{2,q}\gamma_{f;ll} \rightarrow$$

$$M_{2,d} = \frac{1}{8} \cdot g_{2;rep} \cdot \ell_2^2 \cdot \gamma_{f;dl} + \frac{\ell_2^2}{\pi^2} \cdot a\gamma(\hat{d}_{2\,hw} + \hat{\delta}_{2end}) \cdot \gamma_{f;ll} \rightarrow$$

$$M_{2,d} = \frac{1}{8} \cdot 1.663 \cdot 10^2 \cdot 1.2 + \frac{10^2}{\pi^2} \cdot 5 \cdot 10 \cdot 0.361 \cdot 1.3 \rightarrow$$

$$M_{2,d} = 24.9 + 237.8 = 262.7 \text{ kNm}$$

This bending moment results in the following elastic bending stress in the purlins:

$$\sigma_2 = \frac{262.7 \cdot 10^6}{1160 \cdot 10^3} = 226 \text{ N/mm}^2 < \text{fyd} = 235 \text{ N/mm2}$$

The bending stress in the main girders is greater than the yield stress which means that the main girders are not strong enough in case of elastic design. This can be solved by either allowing for plastic design or choosing a heavier section. The bending stresses in the purlins are sufficiently low.

4.2.3 Discussion

In the example above, the application of the set of equations (41) and (44) is illustrated for design and calculation purposes. The set of equations makes it possible to include the interaction between main girders and purlins in a roof structure and also to include the effect of rainwater ponding. In that case, it turns out that the main girders of the example do not have sufficient safety against ponding. In [3], also an example is given where the interaction between main girders and purlins is neglected when calculating for ponding. Then, both main girders and purlins turn out to be safe enough against ponding. This shows that the interaction effect between main girders and purlins cannot be neglected when designing roof structures for rainwater ponding.

5 Sensitivity to construction inaccuracies

The height of the emergency drain opening has a strong influence on the water load: an absolute small increase of the height d_{hw} may give rise to failure. Moreover, the load case may change with increasing height d_{hw} from e.g. load case D to B to C (Figure 2). Also an increase of water level caused by too little slope of the roof may have a strong influence. In this section, a sensitivity analysis is carried out to find out the influence of design and construction inaccuracies on ponding. This is done for relative simple load cases of rigidly supported beams for variations in slope and height of the emergency drain.

*5.1 **Principles for the sensitivity analysis***

The sensitivity analysis is limited to consequences of construction mistakes leading to deviation of the design values for height of emergency drains and roof slope. It is assumed that other deviations leading to other water loads are negligibly small and that no mistakes in modelling and calculation have been made. For an assumed deviation, the increase of the water load is determined, after which the load factor $\gamma_{f;q}$ can be calculated necessary to cover this load increase. Both flat roofs and sloping roofs are considered.

For flat and sloping roofs, the variation in height of the emergency drains Δd_{hw} is considered. Two values for Δd_{hw} are chosen: 5% and 10% of d_{hw} (see Figure 20). For an emergency drain height between 50 and 200 mm, the absolute value of the deviation will thus be between 2.5 and 20 mm.
For flat roofs the deviation from the horizontal level is considered, introducing an adjusting error Δ for the level of the supports (see Figure 21).
For sloping roofs the deviation $\Delta\alpha$ related to the design value of the sloping angle α is introduced (see Figure 24). For $\frac{\Delta\alpha}{\alpha}$ two values are chosen: $\frac{1}{5}$ and $\frac{1}{10}$.

If the geometry of the water load does not change, the quotient:

$$\theta = \frac{M_0 + \Delta M}{M_0} \tag{47}$$

is constant for a given value of n. The parameter θ is an amplification factor, equal to $\frac{n}{n-1}$, and thus a function of n.

For sloping roofs with a partial triangular load, with $p < 1$, at increase of the water level the geometry of the load will change, so the quotient:

$$\psi = \frac{M_0 + \Delta M}{M_0} \tag{48}$$

for a given n is not constant, but depends on n and p (Table 2). For $p = 1$ it holds that: $\psi = \theta$.

5.2 *Flat roofs*

5.2.1 Variation of height of emergency drain

In Figure 20 the variation of the height of the emergency drain is shown as Δd_{hw}.

For an increase of Δd_{hw} it holds that:

$$\gamma_{f;ll} = \frac{M_{end;(d_{hw}+\Delta d_{hw})}}{M_{end;(d_{hw})}} = \frac{C_{m;M_{end}} a\gamma (d_{hw} + \Delta d_{hw})\ell^2}{C_{m;M_{end}} a\gamma\, d_{hw}\ell^2} = \frac{d_{hw} + \Delta d_{hw}}{d_{hw}} \tag{49}$$

For 5% and 10% variation of the height of the emergency drain the following is obtained respectively:

$$\Delta d_{hw} = 0.05 d_{hw} \rightarrow \gamma_{f;ll} = \frac{d_{hw} + \Delta d_{hw}}{d_{hw}} = \frac{d_{hw} + 0.05 d_{hw}}{d_{hw}} = 1.05$$

and

$$\Delta d_{hw} = 0.1 d_{hw} \rightarrow \gamma_{f;ll} = \frac{d_{hw} + \Delta d_{hw}}{d_{hw}} = \frac{d_{hw} + 0.1 d_{hw}}{d_{hw}} = 1.10$$

Table 2: Amplification factor $\psi = \frac{M_o + \Delta M}{M_o}$ *dependent on n and p*

p	n = 1.0	n = 1.25	n = 1.5	n = 2	n = 4	n = 6	n = 8	n = 10
1.0	-	4.76	2.95	1.95	1.32	1.19	1.14	1.11
0.8	-	4.54	2.79	1.88	1.30	1.17	1.12	1.10
0.6	25.33	3.31	2.02	1.58	1.17	1.11	1.08	1.06
0.4	1.79	1.52	1.21	1.15	1.07	1.05	1.03	1.03
0.2	1.09	1.09	1.02	1.02	1.00	1.00	1.00	1.00

Remark: By using numerically calculated coefficients, especially for small values of n, there will be small deviations from analytically calculated values.

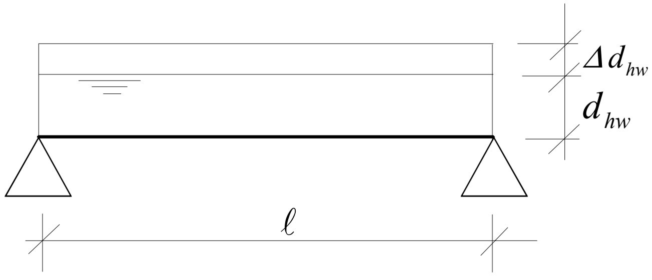


Figure 20: Flat roof – variation of the height of the emergency drain Δd_{hw}

Thus, the larger the imperfection, the larger the load factor has to be to cover up the load increase. In this case the required load factor is smaller than the load factor prescribed in [1], being $\gamma_{f;ll} = 1.3$, so variations in height of the emergency drains of 5% and 10% are covered by the load factor of the code [1]. In the present case a variation of 30% will be covered using a load factor of 1.3, assuming a correct modelling and calculation method.

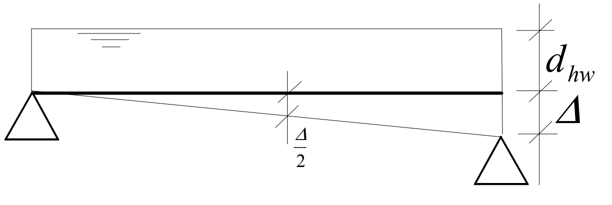


Figure 21: Flat roof – variation of roof slope by adjusting error Δ

5.2.2 Variation of roof slope

In Figure 21 the adjusting error of the support level is given as Δ. The average increase of d_{hw} over the length of the beam is equal to $\frac{\Delta}{2}$. This results in:

$$\gamma_{f;ll} = \frac{M_{end;(d_{hw}+\frac{\Delta}{2})}}{M_{end;(d_{hw})}} = \frac{C_{m;M_{end}} a\gamma(d_{hw} + \frac{\Delta}{2})\ell^2}{C_{m;M_{end}} a\gamma\, d_{hw}\ell^2} = \frac{d_{hw} + \frac{\Delta}{2}}{d_{hw}} = 1 + \frac{\Delta}{2d_{hw}} \tag{50}$$

If d_{hw} and $\gamma_{f;ll}$ are known, with eqn. (50) the maximum allowable value of the adjusting error Δ can be calculated. Alternatively, Δ can be taken from Figure 22. The results are presented for $\gamma_{f;ll} = 1.5$ (safety class 3), $\gamma_{f;ll} = 1.3$ (safety class 2) and $\gamma_{f;ll} = 1.1$ (safety class 1) according to [1].

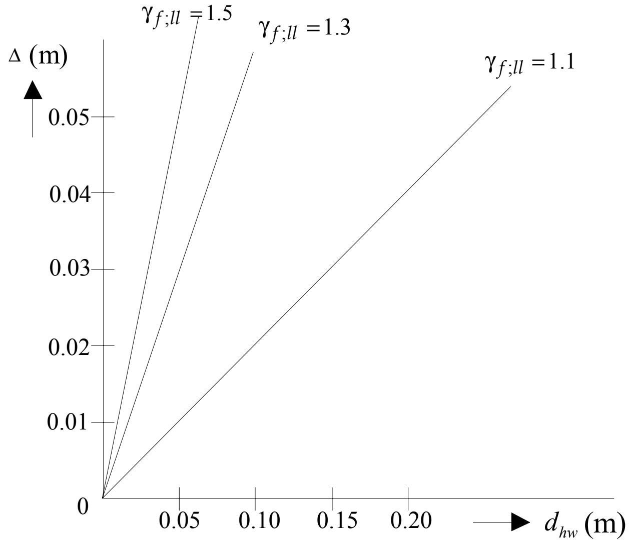


Figure 22: Maximum value of Δ for a given d_{hw} and $\gamma_{f;ll}$

For example, for $\gamma_{f;ll} = 1.3$ and in case $d_{hw} = 50$ mm then:

$$\gamma_{f;ll} = 1 + \frac{\Delta}{2d_{hw}} = 1 + \frac{\Delta}{2 \cdot 50} = 1.3$$

or

$$\Delta = (1.3 - 1) \cdot 2 \cdot 50 = 30 \text{ mm}$$

The adjusting error Δ may not be larger than 30 mm or 60% of the water level at the roof edge (which is the emergency drain height).

5.2.3 Discussion of results

For flat roofs it can be concluded that:

- a 30% variation in the height of the emergency drain is covered by the commonly used load factor 1.3;
- relatively large adjustment errors, 60% of the emergency drain height, are covered by the load factor 1.3.

Since the variations covered by the load factor are relatively great, ponding problems due to construction inaccuracies for flat roofs are expected to be limited.

5.3 *Sloping roofs*

5.3.1 Variation of height of the emergency drain

In Figure 23 the variation of the height of the emergency drain is given as Δd_{hw} .

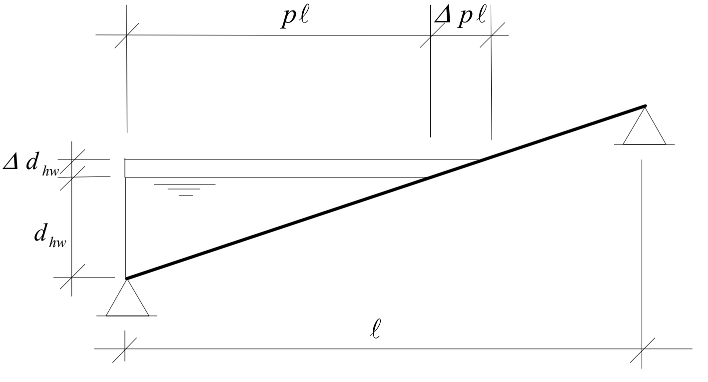


Figure 23: Sloping roof – variation of the height of the emergency drain Δd_{hw}

With a variation of the water level d_{hw}, the size of the triangular load changes. With an increase of the height of the emergency drain by Δd_{hw} , the active width of the water load increases by $\Delta p \cdot \ell$. From the geometry in Figure 23 it follows that:

$$\frac{\Delta p\ell}{p\ell} = \frac{\Delta d_{hw}}{d_{hw}} \tag{51}$$

and thus:

$$\Delta p = p \cdot \frac{\Delta d_{hw}}{d_{hw}} \tag{52}$$

The calculation of the required value of $\gamma_{f;ll}$ will be illustrated by an example.

Assume: $\dfrac{\Delta d_{hw}}{d_{hw}} = 0.1; \;\; p = 0.4; \;\; n = 1.5$.

Then it holds that:

$$\Delta p = p \cdot \frac{\Delta d_{hw}}{d_{hw}} = 0.4 \cdot 0.1 = 0.04 \rightarrow p + \Delta p = 0.4 + 0.04 = 0.44$$

For p = 0.4 Table 1 gives $C_{m;M_{end}} = 0.0212$.

For p = 0.6 Table 1 gives $C_{m;M_{end}} = 0.0690$.

Linear interpolation gives for p = 0.44:

$$C_{m;M_{end}} = \left(0.0212 + \frac{4}{20}(0.0690 - 0.0212)\right) = 0.0308$$

and thus:

$$\gamma_{f;ll} = \frac{M_{end;(d_{hw}+\Delta d_{hw})}}{M_{end;(d_{hw})}} = \frac{0.0308\, a\, \gamma 1{,}1 d_{hw}\, \ell^2}{0.0212\, a\, \gamma\, d_{hw}\, \ell^2} = 1.59$$

This result is shown in Table 3, indicated by the shaded area. More results are presented in Table 3 for 5% and 10% variation in height of the emergency drains for different values of *p* and *n*.

5.3.2 Variation in roof slope

In Figure 24 the variation of the designed roof slope α is indicated as Δα. A variation in roof slope leads to a change of the water load. In case the roof slope α decreases by a value Δa, then the active width of the water load increases by $\ell \cdot \Delta p$.

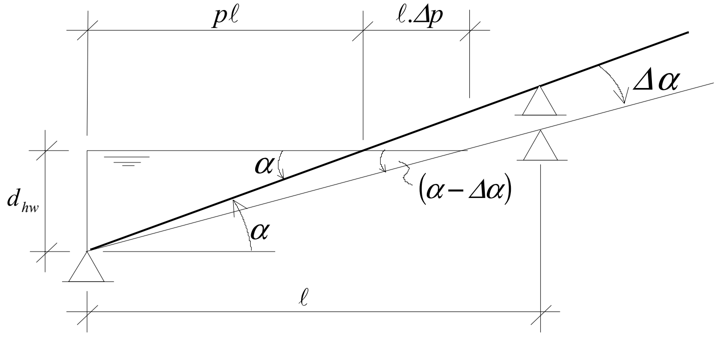


Figure 24: Sloping roof – variation in roof slope Δα

The angles α and Δα are small and thus the following holds: $tan(\alpha - \Delta\alpha) = \alpha - \Delta\alpha$.

From the geometry in Figure 24 it follows that:

$$tan(\alpha - \Delta\alpha) = (\alpha - \Delta\alpha) = \frac{d_{hw}}{(p + \Delta p)\ell} \tag{53}$$

and also the following holds:

$$tan\,\alpha = \alpha = \frac{d_{hw}}{p\ell} \tag{54}$$

From eqns. (53) and (54) it follows that:

$$\Delta p = \frac{p \cdot \Delta\alpha}{\alpha - \Delta\alpha} \tag{55}$$

The value of $\gamma_{f;ll}$ can be calculated using Table 1. This calculation is illustrated by an example again.

Assume: $\alpha = 0.02;\ \Delta\alpha = 0.002;\ n = 1.5;\ \ p = 0.4$

Then the following holds:

$\dfrac{\Delta\alpha}{\alpha}$ = 0.1, and

$$\Delta p = \frac{p \cdot \Delta\alpha}{\alpha - \Delta\alpha} = \frac{0.4 \cdot 0.002}{0.02 - 0.002} = 0.044 \rightarrow \ p + \Delta p = 0.4 + 0.044 = 0.444$$

For p = 0.4 and n =1.5 Table 1 gives $C_{m;M_{end}} = 0.0212$.

For p = 0.44 and n =1.5 Table 1 gives $C_{m;M_{end}} = 0.0308$ by interpolation.

And thus:

$$\gamma_{f;ll} = \frac{M_{end;(\alpha+\Delta\alpha)}}{M_{end;(\alpha)}} = \frac{0.0308\, a\,\gamma\, d_{hw}\,\ell^2}{0.0212\, a\,\gamma\, d_{hw}\,\ell^2} = 1.45$$

This result is shown in Table 3, indicated by the shaded area. More results are presented in Table 3 for $\dfrac{\Delta\alpha}{\alpha} = \dfrac{1}{5}$ and $\dfrac{1}{10}$ for different values of p and n.

5.3.3 Simultaneously varying height of the emergency drain and roof slope

In Table 3, the required partial safety factors $\gamma_{f;ll}$ are also given to cover up for the combined effect of a variation in sill height of $\Delta d_{hw} / d_{hw} = 0.05$ and a variation in roof slope of $\Delta\alpha/\alpha$ = 0.10. These partial safety factors have been calculated in a similar way as indicated above.

5.3.4 Discussion of results

Table 3 gives values for the partial safety factor $\gamma_{f;ll}$ necessary to cover up for construction inaccuracies. According to [1], the partial safety factor is $\gamma_{f;ll} = 1.3$ for safety class 2, which is valid for hall structures with flat roofs. So, for numbers in Table 3 smaller than 1.3, the required safety level is assured and for numbers greater than 1.3, safety is insufficient. The boundary value 1.3 is indicated in Table 3 by underlining the relevant numbers.
Considering the rows 2 and 3 in Table 3 for variation in height of the emergency drain, the influence of n is relatively limited for those cases where $n \geq 1.5$. For $n \geq 1.5$ and a variation

in sill height of 10%, the required partial safety factor exceeds 1.3 in many cases, so this construction inaccuracy is unsafe, especially for small values of p. For a 5% variation in height of the emergency drain, the required safety level is reached for $n \geq 1.5$.

Considering the rows 4 and 5 in Table 3, then for $n \geq 1.5$ the variation in roof slope should meet the requirement $\Delta\alpha / \alpha \leq 0.10$ to almost reach the required safety level corresponding to $\gamma_{f;ll} = 1.3$. However, even then there are cases ($p \leq 0.4$ and $n \leq 2$) where the required safety level is not reached. The influence of n is limited for those cases where $n \geq 1.5$. A greater inaccuracy in roof slope than 10% leads to required partial safety factors far greater than 1.3.

Considering row 6 in Table 3 for the combined variation of sill height and roof slope ($\Delta d_{hw} / d_{hw} = 0.05$ and $\Delta\alpha / \alpha = 0.10$), the required partial safety factor exceeds in many cases 1.3. To cover up for these realistic construction inaccuracies a partial safety factor $\gamma_{f;ll} = 1.8$ is even necessary when n is limited to $n \geq 1.5$. Again, for $n \geq 1.5$ the influence of n is relatively small. However, for $n < 1.5$ the sensitivity to construction inaccuracies is substantial in such a way that even a partial safety factor of 2.0 is insufficient. This being impractical, it is suggested to limit n to $n \geq 1.5$.

6 Conclusions

This article deals with the load case of rainwater ponding on roof structures consisting of rigidly and flexibly supported beams. For rigidly supported beams, a number of load cases are analysed. Also flexibly supported roof beams, namely purlins on main girders, are analysed in this article. Calculation methods are given to design roof structures considering water ponding, without the necessity to use a complex iterative analysis. For roof structures consisting of purlins on main girders, a set of equations is derived which enables the design and calculation for ponding of these structures.

Based on the calculations made and the sensitivity analyses for variations in height of the emergency drain and/or in roof slope, the following conclusions can be drawn:

- The interaction between main girders and purlins always needs to be considered in calculations. If not, the load case water ponding will be underestimated.

Table 3: Required partial safety factor $\gamma_{f;ll}$ for different values of n and p depending on variations in height of the emergency drains Δd_{hw} and roof slope $\Delta\alpha$

	p	n = 1.0	n = 1.25	n = 1.5	n = 2	n = 4	n = 6
$\Delta d_{hw} / d_{hw} = 0.10$	0.8	-	1.24	1.25	1.24	1.22	1.22
	0.6	-	1.44	1.44	1.35	1.31	1.29
	0.4	6.94	1.81	1.59	1.47	1.35	1.33
	0.2	1.69	1.58	1.50	1.47	1.45	1.44
$\Delta d_{hw} / d_{hw} = 0.05$	0.8	-	1.12	1.12	1.12	1.11	1.11
	0.6	-	1.21	1.21	1.17	1.15	1.14
	0.4	3.84	1.39	1.29	1.23	1.17	1.16
	0.2	1.33	1.28	1.24	1.23	1.22	1.21
$\Delta\alpha / \alpha = 1/5$	0.8	-	1.33	1.34	1.32	1.28	1.29
	0.6	-	1.77	1.78	1.56	1.49	1.42
	0.4	14.28	2.62	2.13	1.84	1.57	1.53
	0.2	2.35	2.11	1.90	1.85	1.79	1.77
$\Delta\alpha / \alpha = 1/10$	0.8	-	1.13	1.13	1.13	1.12	1.12
	0.6	-	1.31	1.31	1.23	1.19	1.17
	0.4	6.31	1.65	1.45	1.34	1.23	1.21
	0.2	1.54	1.44	1.36	1.32	1.32	1.31
$\Delta d_{hw} / d_{hw} = 0.05$ and $\Delta\alpha / \alpha = 1/10$	0.8	-	1.26	1.26	1.26	1.23	1.28
	0.6	-	1.54	1.54	1.42	1.37	1.33
	0.4	9.43	2.07	1.76	1.60	1.42	1.39
	0.2	1.90	1.75	1.64	1.60	1.56	1.54

- The variations of the height of emergency drains and the roof slope have large influence on the safety of roof structures for the load case water ponding. A small variation (a higher emergency drain or a smaller roof slope) may result in a lower safety level of the roof structure in case of water ponding, or even in failure of the structure. Therefore, the design values of the height of the emergency drains and the roof slope should be constructed in an accurate way, within limited tolerances.
- The value of the required load factor $\gamma_{f;ll}$ to be used is determined by the shape of the water load, the value of $n = EI / EI_{cr}$ (where EI_{cr} is given by eqn. (14)) and the maximum variation in the roof slope and height of the emergency drain.

- Based on the sensitivity analysis for flat roofs without slope, it can be concluded that a variation in height of the emergency drains of 30% and an adjusting error of the supports of 60% will be covered by a load factor $\gamma_{f;ll}$ =1.3 (relevant for industrial halls [1]). So for flat roofs, problems caused by construction inaccuracies are normally not to be expected.
- Based on the sensitivity analysis for sloping roofs with $n \geq 1.5$ it can be concluded that a load factor $\gamma_{f;ll}$ = 1.8 should be used, while at the same time the variations of the roof slope and the height of the emergency drains should be limited to 10% and 5% respectively. If these tolerances are not feasible or if $n \geq 1.5$, then a load factor even greater than 1.8 is required.
- From the sensitivity analysis it appears that for sloping roofs with $n \geq 1.5$ the influence of the value of n on the safety of the roof structure is small when compared with the influence of construction inaccuracies and the influence of the area covered with water.
- Especially flexible roofs (n < 1.5) are extremely sensitive to construction inaccuracies regarding roof slope and height of emergency drains.
- Based on different considerations in this article and in [3] the authors advise to design roof structures with a value $n \geq 1.5$.
- Further research is recommended on the stochastic distribution of variations of the height of emergency drains and roof slope, in practical situations. With help of these figures and the accepted risks of failure, the required load factors can be calculated.

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