HERON is jointly edited by:
STEVIN-LABORATORY of the
faculty of Civil Engineering,
Delft University of Technology,
Delft, The Netherlands
and
TNO-INSTITUTE
FOR BUILDING MATERIALS
AND STRUCTURES.
Rijswijk (ZH), The Netherlands
HERON contains contributions
based mainly on research work
performed in these laboratories
on strength of materials, structures
and materials science.

ISSN 0046-7316

HERON vol. 33 1988 no. 3

Contents

FATIGUE EVALUATION OF CONCRETE STRUCTURES PRELIMINARY STUDIES, PROCEDURE AND EXAMPLES

A. J. M. Siemes
TNO Institute for Building Materials and Structures

Publication in HERON since 1970

Notations

A list of the symbols used frequently is given below. Symbols used in this report only once have not been included. The meaning of these symbols is explained in the text. Part II (Fatigue strength procedure for concrete structures) contains a separate list of symbols.

c	total number of stress cycles in non-constant-amplitude loading
f_i	frequency
f_{cm}	mean cylinder tensile strength
f'_{cm}	mean cylinder compressive strength
i	index for a certain stress level or frequency
m	number of stress blocks in programme loading test
M_M	Miner number
n_i	number of cycles of a constant-amplitude stress i
N_i	number of cycles giving fatigue failure in a constant-amplitude test at stress level i and frequency i
T	period
β	reliability index
γ_F	load factor
γ_m	material factor
σ_{max}	maximum tensile stress
σ_{min}	minimum tensile stress
σ'_{max}	maximum compressive stress
σ'_{min}	minimum compressive stress

Part I Preliminary studies

1 Introduction

1.1 *Assignment*

The present investigation, commissioned by and carried out in cooperation with the Centre for Civil Engineering Research, Codes and Specifications (CUR), is a logical follow-up of work done previously by TNO-IBBC, the Stevin Laboratory in Delft and the Laboratory Magnel in Ghent in Belgium [1, 2, 3, 4 and 5].
According to schedule the project is closed with a calculation procedure for concrete structures which are subject to fluctuating loads and where fatigue might be a mechanism leading to collapse. It is in the same line with the experimental studies carried out and with recent technological developments to put the procedure on a semi-probabilistic basis. Its ultimate form will be brought into line, as far as possible, with the semi-probabilistic design procedures that have been in use for some time now for concrete structures, mainly subject to static loads. Examples are to be found in the present dutch rules for concrete VB 1974/1984, in the draft dutch building code TGB 1986 and in the Eurocodes.

1.2 *Statement of problem*

Generally speaking, concrete structures are mainly subjected to static loads. In cases of (partially) dynamic loading it is usually quite feasible, by means of an "amplification factor", to treat the dynamic load as an equivalent static load. Such a load should be characterized by occurring only occasionally during the design life of the structure. If, however, a concrete structure is subjected to a load varying strongly in the course of time, collapse due to fatigue cannot be ruled out. This implies that the cumulative damage, due to all the load fluctuations that occurred during the service life of a structure, may be ultimately responsible for its collapse. This turns fatigue into a matter of service life, similar to other durability aspects of concrete, such as chemical attack, corrosion and erosion. In the DNV Rules [8], among other publications, this aspect is reflected by the fact that the fatigue limit state is included in the serviceability limit state. In today's design rules there is an increasing tendency to include fatigue in the ultimate limit state, at least as far as the failure aspect is actually considered. Crack formation, deflection and, more in general, the decrease in stiffness due to fluctuating loads are included in the serviceability limit state.
Apart from the fact that a fatigue assessment is essentially an assessment of service life (and is hence still rather unusual in design rules), there is still another problem. In the case of statically loaded structures, the procedure used to determine the load-bearing capacity in relation to the probability of failure and the safety margin to be applied, is based on decades of experience. This is true both of the calculation procedure and of the implicit outcome of that procedure (the probability of failure).

A calculation procedure can, in principle, be calibrated against the reliability of structures observed in actual practice. In the assessment of fatigue there is practically no such feedback. The types of structures, often big and costly, in which fatigue may play a role are relatively small in number. Consequently, the incidence of fatigue failure in actual practice in quite low; possibly, in a number of cases, it was not even recognized. As a result, quantitative information on the reliability of concrete structures subject to fatigue loading is lacking.

Insight into the material behaviour of concrete structures has been improved considerably by recent research carried out in The Netherlands by StuPOC, CUR and MaTS, among others. Consequently, in the assessment of fatigue, the contribution of material behaviour is now markedly easier to handle. The studies carried out did not aim at a better understanding of the phenomenon of fluctuating loads or the behaviour of structures subject to such loads. In these two areas the studies in question will not basically improve fatigue assessment. It is possible, however, to utilize the insights gained for a better design and dimensioning of structures, thus ensuring an improved correlation between the load and structural aspects, on one hand, and material behaviour, on the other.

1.3 *Summary of MaTS and CUR studies of plain concrete*

The MaTS fatigue studies linked up with research previously done in this field in The Netherlands, namely the StuPOC studies [1] and the CUR studies [2, 3, 4 and 5]. All this work concerned cyclic concentric compressive loading of plain gravel concrete. The aim of those investigations was to establish whether the effect of loads varying randomly in time was satisfactorily expressed by using Miner's Rule. Nearly all the previous research into fatigue behaviour had been based on experiments with stresses varying sinusoidally with time, called constant-amplitude tests (see Fig. 1). The results are plotted in a Wöhler diagram (see Fig. 2), which relates the number of stress cycles, N_i, to the maximum stress recorded.

In the StuPOC investigations loads were programmed (see Fig. 3), the specimen being subjected to successive periods (blocks) of different constant-amplitude stresses, which produced a certain measure of random loading. In the CUR studies referred to, another

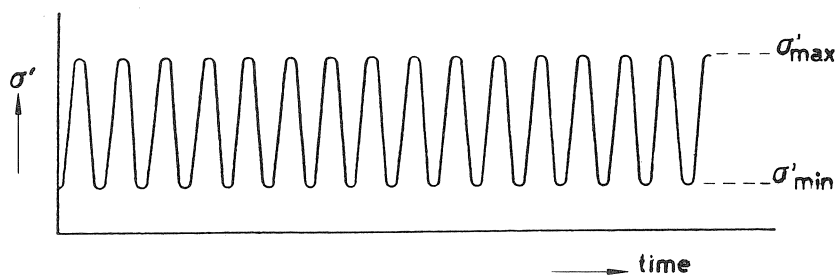


Fig. 1. Constant-amplitude loading.

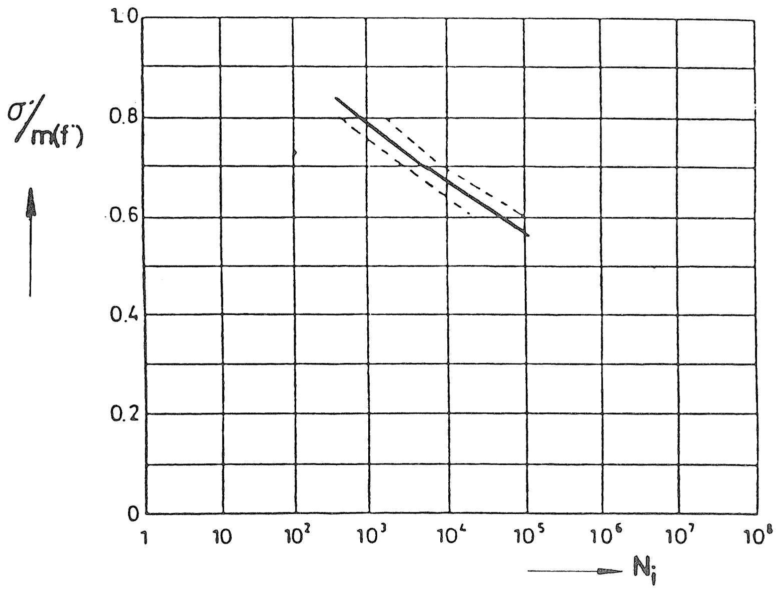


Fig. 2. Wöhler diagram.

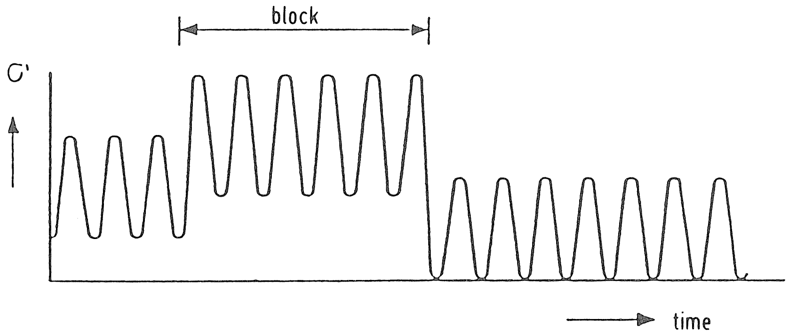


Fig. 3. Programmed loading.

step forward was taken by means of variable-amplitude tests (see Fig. 4). These consisted of full or half cycles in which amplitudes were varied all the time. The main conclusion from both studies was that in these cases Miner's Rule is readily applicable. This rule is expressed by the formula:

$$\sum_{i=1}^{n} \frac{1}{N_i} \leq 1$$

where

n = is the total number of stress cycles
N_i = the number of stress cycles at stress level i at which, in a constant amplitude test, fatigue failure would occur

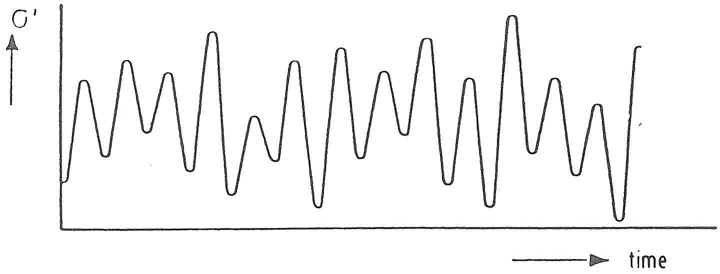


Fig. 4. Variable amplitude loading.

In the interpretation of the test results due allowance had to be made for the spread in fatigue life. Therefore, it is formally more correct to replace the value "unity" in the right-hand side of the inequality by the quantity M_M (the so-called Miner number). The mean value of M_M generally deviates to a limited extent from unity.
It was found that, in spite of the random mode of loading, the scatter was not greater in programme-loading and variable-amplitude tests than in constant-amplitude tests.
Fatigue life proved to be dependent on:
- maximum stress;
- minimum stress;
- cycle frequency;
- grade of concrete;
- curing time;
- curing and testing conditions (under or above water).

As a follow-up of these studies, fatigue research has been carried out at the Stevin Laboratory of the Delft University of Technology and at the Magnel Laboratory of the Ghent State University, based on the generation of cyclic tensile stresses and of stresses alternating continuously between tension and compression. At the Stevin Laboratory concentric tests were carried out and at the Magnel Laboratory flexure tests. To begin with, constant-amplitude tests were performed. The results have enabled the Goodman diagram for compression (see Fig. 5) to be extended to include the tension part and the tension/compression part. In the discussion of the results of the constant-amplitude tests a distinction is made between cyclic tension tests, alternating tension/compression tests with failure in tension, alternating tension/compression tests with failure in compression and cyclic compression tests. The experiments in Delft were carried out with dry and wet B 45 grade concrete. In a number of cases the concrete was tested under water. The frequency applied in concentric loading tests was in general 6 Hz. From the results of the various tests general expression for the Wöhler curves have been derived, using regression analysis:

Cyclic tension tests (dry)

$$\log N_i = 14.81 - 14.52\sigma_{max}/f_{cm} + 2.79\sigma_{min}/f_{cm}$$

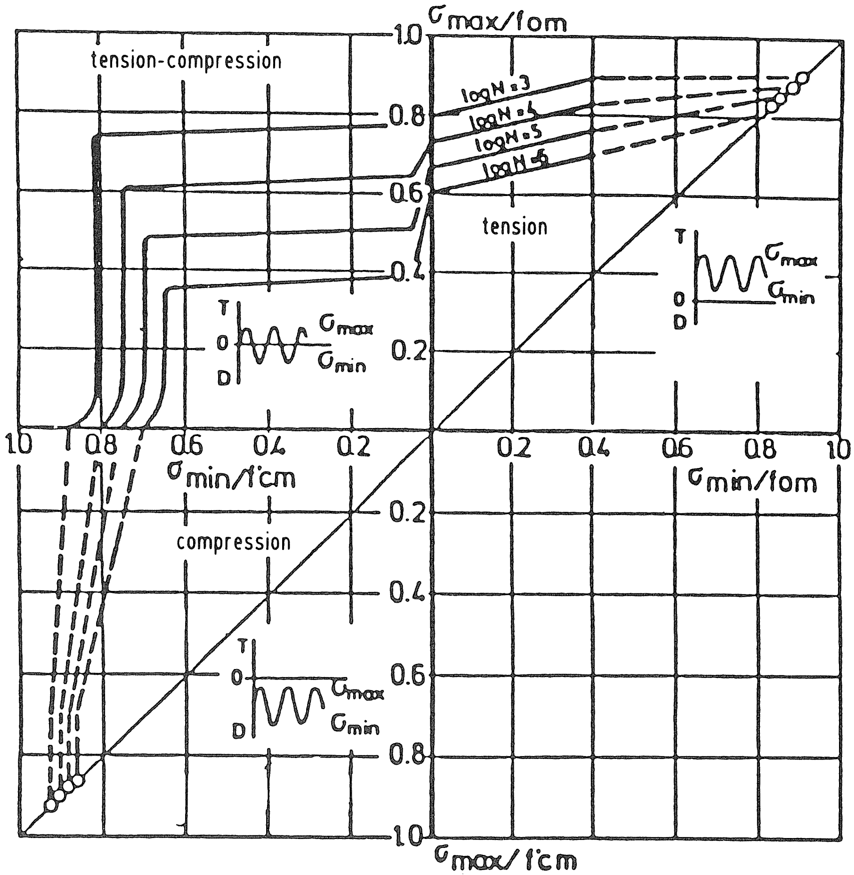


Fig. 5. Goodman diagram.

Cyclic tension tests (wet)

$$\log N_i = 13.92 - 14.52\sigma_{max}/f_{cm} + 2.79\sigma_{min}/f_{cm}$$

Alternating tension/compression tests with failure in tension

$$\log N_i = 8.94 - 7.68\sigma_{max}/f_{cm} + 0.37\sigma_{min}/f_{cm}$$

Alternation tension/compression tests with failure in compression

$$\log N_i = 1.58(\sigma'_{min}/f'_{cm})^{3.14} \quad \text{for} \quad \sigma_{max} > 0$$

Cyclic compression tests

$$\log N_i = 2.00(\sigma'_{min}/f'_{cm})^{3.14} \quad \text{for} \quad \sigma_{max} = 0$$

When in the alternating tension/compression tests extreme compressive stresses exceeded approximately 65 % of the compressive strength, failure occurred in compression. The combined results are presented in the Goodman diagram of Fig. 5. At values of $N_i > 1000$ alternating tensile/compressive stresses lead to a relatively shorter fatigue life than cyclic tensile stresses. During the tests not only the number of cycles to failure was recorded, but also the longitudinal deformations. From the results it was concluded that there is a distinct relationship between the number of cycles to failure and the (cyclic) rate of deformation $\mathring{\varepsilon}_{sec}$ (i.e. the increase in deformation per unit time occurring at the maximum stress). The influence of a number of external factors on service life has also been examined:

- moisture content; wet specimens have shorter lives than dry ones loaded to the same stress level relative to the static strength;
- water penetration; the penetration of water into "cracked" or "non-cracked" concrete was found to have no effect;
- frequency; a lower frequency lowers the number of cycles to failure, but not proportionally;
- light weight aggregate; no significant difference was found between dry light weight concrete and dry gravel concrete, but this need not necessarily be true for all types of light weight aggregate.

By means of programme-loading tests and variable-amplitude tests the applicability of Miner's Rule has been investigated. The test results showed a considerable scatter. When, however, the value of N_i was estimated on the basis of the rate of secondary strain, the scatter decreased and Miner's Rule could be applied with sufficient confidence. Miner's Rule does not supply any information on the changes in strength and stiffness during service life. Such information is of importance in assessing the deformations and the static strength in any phase of service life. Therefore, static tests have been carried out in which the strength and the modulus of elasticity were measured. The specimens had previously been subjected to cyclic loads for periods of between 20 and 100 % of the calculated fatigue life. It was found that in particular the stiffness decreased considerably, especially during alternating tensile/compressive loads. The modulus of elasticity dropped to 40–80 % of its original value.

The influence of the stress gradient across the section was examined in comparative studies at the Magnel Laboratory. The experiments were carried out on beams of which one part was used to determine the static strength and another part was then subjected to a constant-amplitude fatigue test.

From the results of these tests the following equations were derived:

Cyclic tension tests

$$\log N_i = 14.61 - 13.78\sigma_{max}/f_{cm} + 2.24\sigma_{min}/f_{cm}$$

Alternating tension/compression tests with fracture in tension

$$\log N_i = 9.91 - 7.45\sigma_{max}/f_{cm} + 1.93\sigma_{min}/f'_{cm}$$

When these results are compared with the concentric tests carried out in Delft, flexure proves to have a favourable effect on fatigue life. This is true in particular for alternating tensile/compressive stresses with large amplitudes. This phenomenon can be explained by a possible redistribution of stresses due to the presence of a stress gradient, but it will have to be examined more closely.

During the TNO-IBBC studies, extensive experiments have been carried out to establish whether in the case of random-type stresses Miner's Rule is still applicable. In the first series of experiments variable-amplitude stresses were generated in concentrically loaded concrete. Later the work was extended to include various forms of random loading. In order to verify Miner's Rule over the widest possible range of conditions, the tests were carried out using different grades of concrete, after various periods of curing. In addition, curing and testing conditions were varied.
Since an accurate calculation of Miner numbers is possible only when reliable results of constant-amplitude tests are available, such tests were invariably included in the studies. From the outcome it was concluded that, at a certain relative stress level, a higher static compressive strength of the concrete will lead to a shorter fatigue life than a lower strength. The way in which the higher strength is attained (curing time, mix composition, curing conditions) does not make much difference in this connection. It was further found that a lower cycle frequency leads to a lower number of cycles to failure. But the drop in the latter figure is less than proportional, so that fatigue life (expressed in units of time) is, in fact, somewhat longer.
From the experimental data the following average relations have been derived for the Wöhler curves. In repeat tests small deviations from these results were observed:

Grade 45 concrete, cured under water and 28 days old

$$\log N_i = -\frac{14.2 \cdot \log \sigma'_{min}/f'_{cm}}{\sqrt{(1-R_i)}} + 1.6$$

Grade 45 concrete of a different composition, cured under water and 28 days of curing

$$\log N_i = -\frac{14.2 \cdot \log \sigma'_{max}/f'_{cm}}{\sqrt{(1-R_i)}} + 1.6$$

Grade 45 concrete, cured under water and six months of curing

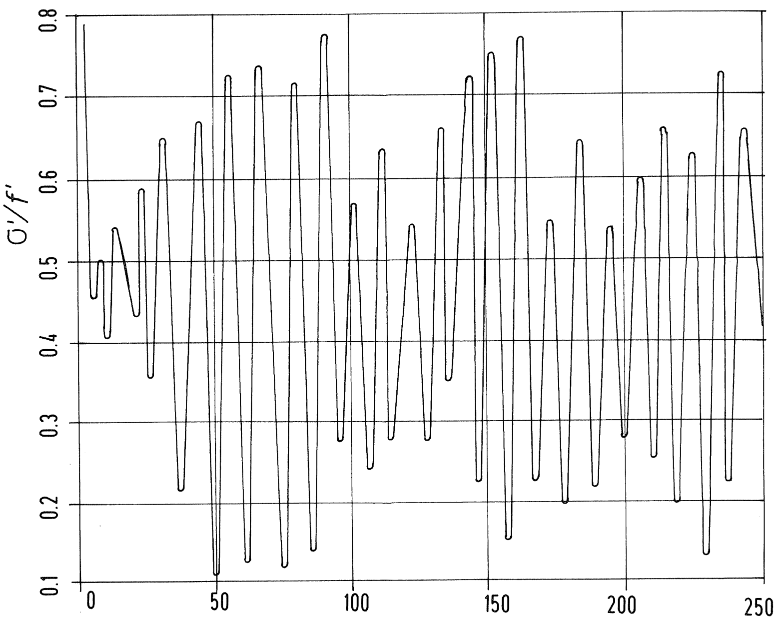
$$\log N_i = -\frac{21.3 \cdot \log \sigma'_{max}/f'_{cm}}{\sqrt{(1-R_i)}} + 0.5$$

Grade 30 concrete, cured under water and 28 days of curing

$$\log N_i = -\frac{13.2 \cdot \log \sigma'_{max}/f'_{cm}}{\sqrt{(1-R_i)}} + 1.9$$

Grade 45 concrete, cured at 20°C en 65 % R.H. and 28 days of curing

$$\log N_i = -\frac{17.0 \cdot \log \sigma'_{max}/f'_{cm}}{\sqrt{(1-R_i)}} + 2.0$$



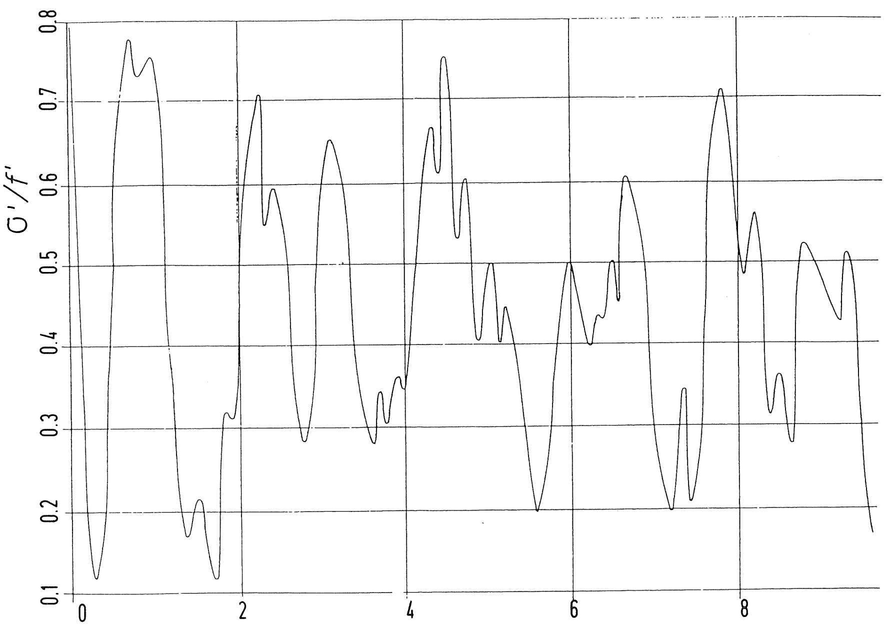


Fig. 6. Some examples of stationary random loading.

Following the studies with variable-amplitude stresses, various types of random loading were applied (see e.g. Fig. 6):

- stationary random stresses, with variations in distribution and spectrum;
- non-stationary random stresses, the mean of the distribution being kept constant;
- non-stationary random stresses with several rest periods between periods of otherwise stationary random stresses;
- non-stationary random stresses, the mean being varied after a certain period of time.

In all these cases Miner's Rule was found to be readily applicable. Or on average the value found for the Miner number was approximately equal to unity, the scatter generally even being smaller than was expected in view of the scatter in the results of the constant-amplitude tests. In the case of random loads with spectra in which no main frequency could be distinguished, an unequivocal determination of the number and the amplitude of cycles is not possible. For this reason a certain counting method is proposed, called the TNO counting method. It distinguishes between:

- the maximum stress;
- the minimum stress;
- the frequency.

Small fluctuations in the stress are neglected; fluctuations are distinguished only when a special condition is fulfilled, namely that the stress should first have reached the mean value. Fig. 7 gives an example of this counting method.

In addition to concentric compression tests, eccentric compression tests have been performed. In such tests the maximum stress occurs in only one single fibre, all the other fibres being loaded less heavily. It may be expected therefore that, as a result of a

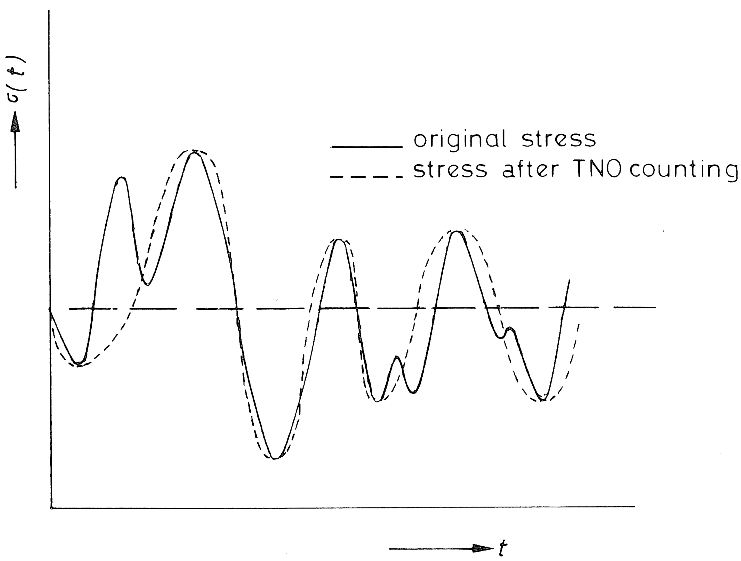


Fig. 7. Example of counting by the TNO method.

redistribution of stresses over the section, the situation will be more favourable than under a concentric compressive load. Fatigue life might therefore be longer. This hypothesis was verified in experiments with various types of concrete, curing conditions, curing times and types of random loading. The results furnished no conclusive evidence. The effect is too small to be taken into account in the design.

2 Review of existing design procedures

2.1 *General*

In the following review of the various design rules we have, in principle, adopted the notation of symbols used in the rules concerned. The construction of large concrete offshore structures in the North Sea in the last decade has led to the formulation of several sets of design rules for concrete structures that might show fatigue phenomena due to fluctuating loads. The first guidelines were more or less born of need. They were based mainly on studies relating to types of concrete and test conditions and not sufficiently geared to the conditions prevailing in offshore structures. Nevertheless, those guidelines do take into account the basic material behaviour, so that presumably proper allowance is made for the fatigue aspect. The knowledge acquired more recently should make it possible to come to more accurate calculation procedures and especially to a better assessment of their reliability.

To give an idea of the state of the art with respect to calculation procedures for concrete structures subject to fatigue loading, we shall now briefly review the most important design rules. The review, rather than trying to be complete, is an attempt to give the broadest possible picture. The following will be discussed:

- Rules for Concrete 1974/1984 (VB 1974/984) [7] (in Dutch);
- Rules for the Design, Construction and Inspection of Offshore Structures 1977 of Det Norske Veritas (DNV Rules [8]);
- Regulation for Structural Design of Load-bearing Structures Intended for Exploitation of Petroleum Resources 1985, of the Norwegian Petroleum Directorate (NPD Regulations [9]);
- Recommendations for the Design and Construction of Concrete Sea Structures, of the FIP, 1985 (FIP Recommendations [10]);
- the TNO-IBBC procedure of 1975 [16];
- the VNC method for plain-concrete roads [6] (in Dutch);
- the Draft Rules of Concrete Bridges, version of January 1988 [23] (in Dutch);
- GTG 15 Fatigue of Concrete Structures [25].

2.2 *Rules for Concrete 1974/1984 [7]*

These rules contain a minimum of information on the aspect of fatigue. With respect to plain concrete (article D-103.1), reinforced concrete (article E-103.1) and prestressed concrete (article F-103.1), the field of application of the rules is restricted to concrete

structures in which the influence of fatigue is only slight or negligible. The rules do not indicate how this condition can be shown to be fulfilled.
In the discussion of article F-103.1 it is stated, however, that road bridges and railway viaducts should be regarded as structures in which the influence of fatigue is small, but not negligible. In the discussion of article E-304.2 the reader is referred to the CUR reports "Strength and stiffness of columns under alternating load" [26 and 27] (in Dutch).
To take this influence into account, the prestressing steel and the anchoring systems have to meet the specification of Dutch Standard 3868 and Dutch Standard 3869 respectively. Finally, in the discussion of article E-103.1 it is stated that the influence of fatigue can be taken into account through an equivalent static load. It is not indicated, however, how this equivalence should be determined.

2.3 *DNV Rules 1977 [8]*

First a number of individual criteria is given by which it can be determined whether or not fatigue may be ignored. Fatigue is negligible if:
- no tension occurs and the design stress is not higher than half the design strength, or:
- the number of stress cycles does not exceed 10,000, or:
- the design stress range S_r corresponding with 10,000 cycles is smaller than the fatigue strength f_r at $2 \cdot 10^6$.

For concrete in compression and flexure and in shear, relations are given for f_r. Roughly speaking, these relations state that f_r is smaller than $0.4 f_k$ in pure flexure and smaller than $0.5 f_k'$ in pure compression, where f_k' is the characteristic compressive cylinder strength. The values specified may be lower in underwater applications and when higher maximum stresses occur during load cycles.
If fatigue has to be taken into account, this is done at the level of constant-amplitude stresses.
To describe fatigue strength at constant-amplitude stresses, use is made of a linear relation between the maximum stress σ' and log N_i (Wöhler curve). The influence of the minimum stress is expressed in a linear Goodman diagram. This leads to the following expression:

$$\log N_i = C_2 \frac{1 - \sigma'_{max}/f'_d}{1 - \sigma'_{min}/f'_d}$$

The design compressive strength of the concrete then follows from:

$$f'_d = \frac{\alpha \cdot f'_k}{\gamma_m}$$

where:

α = factor varying from 1.0 for pure compression to 1.26 for pure flexure
γ_m = material factor, taken to be 1.25
f'_k = characteristic compressive cylinder strength

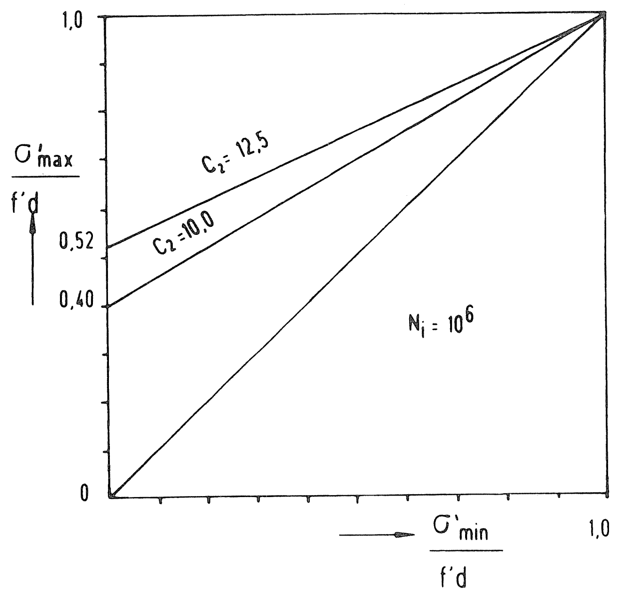


Fig. 8. Example of a Goodman diagram according to [8].

Constant C_2 is dependent on whether the concrete is under water ($C_2 = 10.0$) or above water ($C_2 = 12.5$). By way of illustration, a Goodman diagram for $N_i = 10^6$ and for $C_2 = 10.0$ and $C_2 = 12.5$ is given in Fig. 8. An example of a Wöhler curve is given in Fig. 9. The curve in question holds for $\sigma'_{min} = 0$ and for $C_2 = 10.0$ and $C_2 = 12.5$.
To take into account the influence of non-constant-amplitude stresses, a modified Miner's Rule is used:

$$\sum_{i=1}^{m} \frac{n_i}{N_i} \leq 0.2$$

where:

- m = number of blocks (at least 8)
- n_i = number of stress cycles in block i during the design service life
- N_i = number of stress cycles giving fatigue failure at the mean stress and the amplitude of block i

In the case of stresses varying randomly with time, the number of cycles and their amplitude must be counted using a counting method simular to that of TNO. A scientific basis for this method is not given. A striking aspect is that a stress cycle is defined by the minimum stress, the maximum stress and the period T (or the frequency $1/T$). The dependence of fatigue strength on frequency, however, is left undiscussed. The same applies to the stochastic and log-normal character of the fatigue process. In view of the log-normal character, it would be definitely wrong to translate the value of 0.2 into a safety factor of $1/0.2 = 5$.

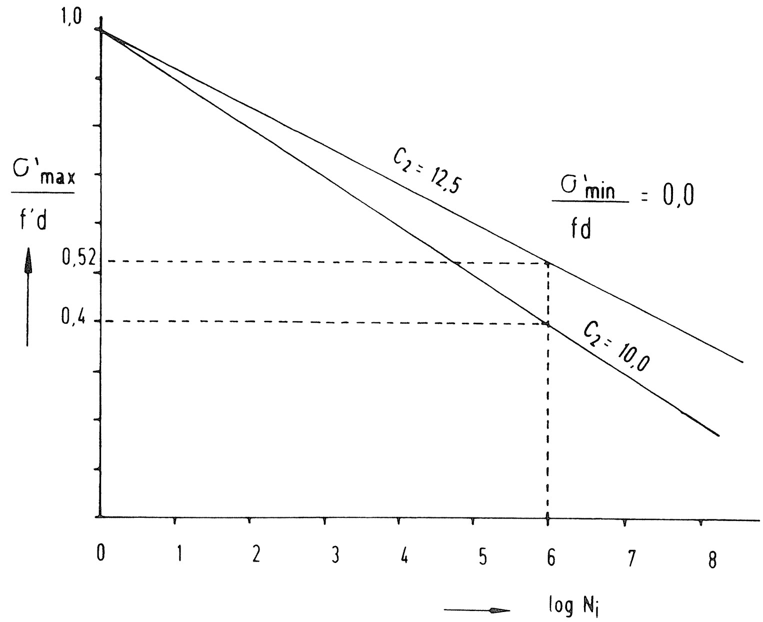


Fig. 9. Example of a Wöhler diagram according to [8].

2.4 *NPD Regulations 1985 [9]*

The NPD Regulations are based on an ultimate limit state. The calculation should then be based on the fatigue load expected to occur during the service life of the structure under consideration.
Dependent on the importance of the structure, on one hand, and on the accessibility for inspection and repairs, on the other, the number of stress cycles has to be multiplied by a factor varying between 1 and 10. The material factor to be used is 1.0. To assess the fatigue aspect, use is made of Miner's Rule:

$$\sum_{i=1}^{m} \frac{n_i}{N_i} \leq 1.0$$

where:

m = number of stress blocks
n_i = number of stress cycles in stress block i
N_i = number of stress cycles to fatigue under stress conditions corresponding with those of stress block i

For concrete loaded in compression, tension or shear or for the bond zone, a reduced design strength f_r has to be employed equal to:

$$f_r = f(1 - 0.08 \cdot \xi \cdot \log N_i)$$

where

$\xi \ = \Delta\sigma/f_r \leq 1.0$

$\Delta\sigma =$ change in stress when the load changes from its maximum to its minimum value

$N_i \ =$ number of stress cycles leading to fatigue failure

$f_r \ =$ strength of concrete (compression, shear or bond)

The NPD Regulations must be regarded as incomplete. They begin by recommending the use of Miner's Rule, but fail to state how the corresponding Wöhler curves can be obtained. Neither do they give any counting method. Furthermore, the regulations revert to a method with reduced strengths.
As regards the Miner number of 1.0, a simular remark can be made as in section 2.3 about the value of 0.2. Here it should be stated that the value of 1.0 does not mean that the calculation procedure is based on a lower safety level than, for example, the DNV rules.

2.5 *FIP Recommendations 1985 [10]*

Part of the FIP Recommendations is also devoted to the fatigue limit state. Assessment takes place at several levels of complexity, to be passed through in succession:
- check whether the number of stress cycles remains below the limit of 1000;
- check whether the stress remains below a certain limiting value; for concrete loaded in compression this value is equal to $0.5f'_{ck}$, where f'_{ck} is the characteristic compressive cylinder strength after 28 days curing; in f'_{ck} half of the curing effect may be taken into account;
- complete fatigue analysis.

Here, too, complete fatigue analysis is based on Miner's Rule:

$$\sum_{i=1}^{e} \frac{1}{N_i} < 1$$

where

$e \ =$ expected ($P = 50$ %) number of stress cycles during service life

$N_i =$ the maximum number of constant-amplitude stress cycles of the same magnitude as the stress cycle under consideration

The value of N_i should be deduced from characteristic Wöhler curves (viz. the $\mu - 2\sigma$ curves) in which a material factor γ_m has been taken into account. Apparently, these curves have to be determined anew each time. The Recommendation emphasize the influence of cycle frequency. With regard to the Miner number 1.0 the same remark holds as in 2.3.

2.6 *TNO-IBBC procedure 1975 [16]*

For the purpose of evaluating the designs of several concrete offshore structures, TNO-IBBC developed a procedure for the assessment of fatigue strength in 1975.

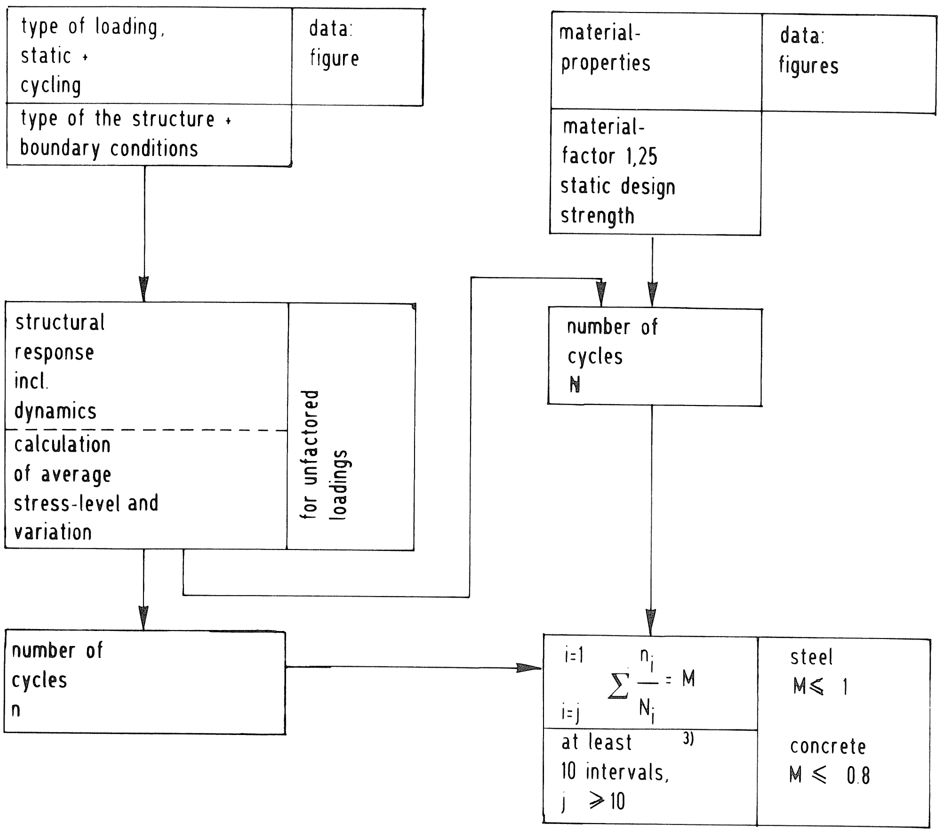


Fig. 10. Diagram of TNO-IBBC design procedure [16].

In view of the lack of knowledge existing at the time especially with respect to tensile strength and load strength under cyclic loading, the field of application of the procedure is restricted to:

- reinforced concrete elements under relatively low cyclic loading;
- prestressed concrete elements in which under service conditions no tensile stresses occur and in which under ultimate conditions the crack width remains below 0.1 mm.

The procedure comprises both load and material data. It is shown in diagrammatic form in Fig. 10. By way of illustration, Fig. 11 shows the Wöhler curve for $\sigma'_{\min} = 0$ and Fig. 12 the Goodman diagram for $N_i = 2 \cdot 10^6$ load cycles.

The Wöhler curve has two striking features:

- a restriction of the maximum fatigue strength to 0.9 × the design value of the static strength;
- a fatigue limit at 0.5 × the design value of the static strength.

To take into account the effect of non-constant-amplitude stresses, TNO too, makes use of Miner's Rule:

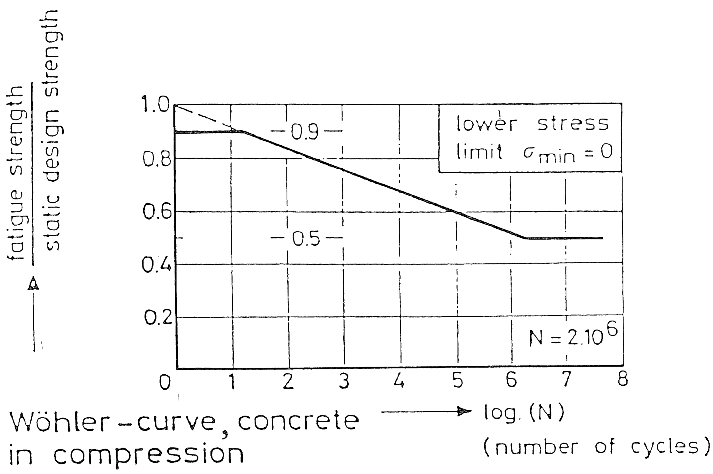


Fig. 11. Example of Wöhler curve from [16].

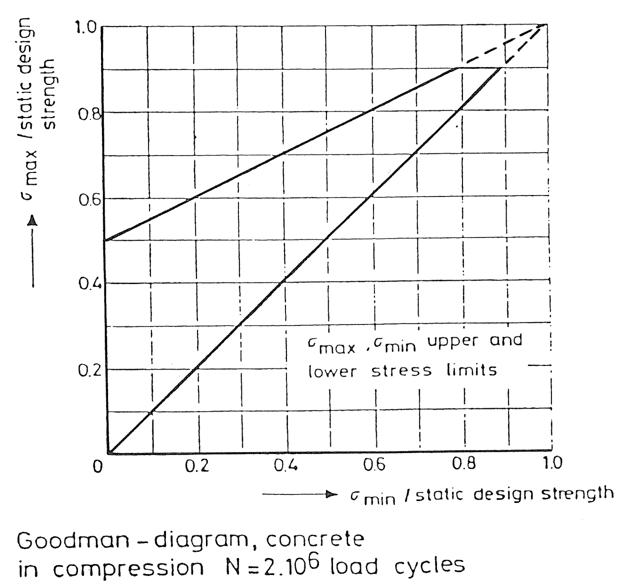


Fig. 12. Example of Goodman diagram from [16].

$$\sum_{i=1}^{i=j} \frac{n_i}{N_i} \leqq 0.8$$

where

n_i = number of load cycles during service life, with specific values for σ'_{min} and σ'_{max}

N_i = maximum number of load cycles possible at the specific values for σ'_{min} and σ'_{max}

j = number of intervals (at least 10)

The extreme wave should be treated as a separate interval. With regard to the Miner number of 0.8 the same remark can be made as in 2.3.

2.7 *VNC method for plain concrete roads 1980 [6]*

The Concrete Research Foundation of the Dutch Cement Industry (Dutch initials VNC) has devised a method of calculation for plain concrete roads. In [22] this method is commented.
The method is primarily concerned with the determination of the stresses in plain-concrete slabs with dummy joints. These stresses are then due either to wheel loads or to temperature differences through the thickness of the concrete slabs. The method reckons with a transfer of loads at the location of the joints, due to the presence of steel dowels. The critical case of loading occurs, in principle, when an axle load is applied near a longitudinal or transverse joint in the road carpet.
To describe fatigue behaviour when there is no temperature gradient the following model is used:

$$N_{Pi} = 10^{12.6}(1 - r_{Pi})$$

where

N_{Pi} = maximum number of passages of load P_i
r_{Pi} = relative tensile stress in the slab due to load P_i; the following equation holds:

$$r_{Pi} = \frac{\sigma_{Pi}}{f_b}$$

σ_{Pi} = tensile stress in the slab due to load P_i
f_b = characteristic flexural tensile strength of the concrete after 90 days

In the presence of a temperature gradient the model changes into:

$$N_{Pi,\Delta t} = 10^{12.6}\left(1 - \frac{0.8 r_{Pi}}{0.8 r_t}\right)$$

where

$N_{Pi,\Delta t}$ = maximum permissible number of passages of load P_i in the presence of a temperature gradient Δt in the slab
r_t = relative tensile stress as a result of the σ_t; the following equation holds:

$$r_t = \frac{\sigma_t}{f_b}$$

σ_t = temperature stress due to the gradient Δt

To take into account the overall effect of all traffic loading during the design service life, use is made of Miner's Rule. Since one part, ξ, of the wheel loading occurs in the presence of a temperature gradient and another part, $1-\xi$, in the absence thereof, the Rule is formulated as follows:

$$PMG = \sum_{i=1}^{i=n} \left(\frac{\xi n_{Pi}}{N_{Pi,\Delta t}} + \frac{(1-\xi) n_{Pi}}{N_{Pi}} \right)$$

where

n_{Pi} = number of passages of load P_i
n = total number of passages

For the temperature gradient Δt at value of 0.07 °C/mm is taken. On the basis of the fatigue studies carried out by TNO-IBBC, a value of 0.5 is used as the average value of the Miner number. This value is also adopted as the design criterion, so that:

$$PMG \leqq 0.5$$

The calculation procedure does not include a determination of the load spectrum.

2.8 *Draft Rules for Concrete Bridges, version of December 1988 [23]*

Section 5.6 of these rules deals with fatigue. The fatigue aspect should be considered in the light of the frequently passing mobile loads. If the magnitude of these loads is unknown, it may be taken from the figure for mobile loads used in the (semi-) static calculation. In that case a reduction factor may be taken into account. This factor is dependent on the type of bridge and on the class of load. As load/material a value of $\gamma = 1$ has to be used.
The phenomena to be checked are:
- the maximum compressive stress in the concrete under service conditions; the maximum value of the stress cycle should not exceed $0.25 f'_{bk}$;
- the stress cycles in the reinforcement under service conditions; these need not be checked when the stresses due to frequently passing mobile loads are lower than a certain percentage P_{max}. The value of P_{max} is dependent on the type of steel and on the mode of fabrication and varies from 35 N/mm^2 to 220 N/mm^2, unless a Smith diagram yields another value and a dynamic amplification factor and a safety factor $\gamma = 1.15$ have been taken into account;
- in the case of prestressed concrete the stress cycles in the steel need not be checked when the stress cycles are absorbed in the section through the concrete.

2.9 *GTG 15 Fatigue of Concrete Structures 1987 [25]*

These rules give fatigue design procedure of general validity for concrete structures. The procedure has to be used if more than 10,000 stress cycles occur. A distinction is made between ultimate limit states and serviceability limit scatter.

To make allowance for the scatter in material properties. Wöhler curves for $\mu - 1.65\sigma$, the 5 % fractile have been introduced. The procedure can be carried out at three different levels:

- at less than 10^7 stress cycles by limiting magnitude of the maximum stress;
- by a limitation of the maximum stress depent on the number of stress cycles occurring;
- by taking into account the load spectrum using Miner's Rule; the number of cycles may be counted using the rainflow or the TNO counting method; the Miner number is put equal to "1" when in the design Wöhler curve partial safety factors have been incorporated with $\gamma = 1.15$ for steel and $\gamma = 1.25$ for concrete; the Miner number is put equal to "1.0" when using characteristic Wöhler curves; the question of including a partial safety factor is still being considered; the magnitude of that safety factor is not given.

2.10 *Summary*

In the preceding sections some important procedures have been discussed. For the purpose of a numerical comparison, the main parameters from the various procedures will now be reviewed, together with a quantification.

Rules for Concrete 1974/1984 [7]

- contain no relevant information.

DNV Rules 1977 [8]

- load factor $\gamma_F = 1.0$ on the service load during the design service life;
- material factor $\gamma_m = 1.25$ on the characteristic strength;
- no fatigue limit;
- no influence of frequency;
- influence of use of concrete underwater;
- influence of stress gradient taken into account;
- gives counting method;
- Miner number equal to 0.2.

NPD Regulations 1985 [9]

- load factor $\gamma_F = 1.0$ on the service load; dependent on the importance of the element under consideration and on the accessibility to inspection and repairs, service life may vary by a factor of 1 to 10;
- material factor $\gamma_m = 1.0$ on the characteristic strength;
- no fatigue limit;
- no influence of frequency;
- no influence of use of concrete underwater if not cracked;
- no counting method included;
- Miner number equal to 1.0.

FIP Recommendations 1985 [10]

- load factor $\gamma_F = 1.0$ on the service load during the design service life;
- material factor $\gamma_m = 1.25$ on the characteristic strength after 28 days; part of the curing effect may be taken into account;
- fatigue limit is not mentioned, Wöhler curves to be deduced from experimental data;
- the same applies to the influence of frequency;
- ditto for concrete used under water;
- no counting method included;
- Miner number equal to 1.0.

TNO-IBBC procedure 1975 [16]

- load factor $\gamma_F = 1.0$, the wave loads being specified;
- material factor $\gamma_m = 1.25$ on the characteristic compressive cylinder strength after 28 days; the curing effect must be taken into account;
- fatigue limit at $N_i = 2 \cdot 10^6$ (for $\sigma'_{min} = 0$ at $\sigma'_{max} = 0.5 f'_c$);
- maximum stress limited at $0.9 f'_c$;
- no influence of frequency;
- no influence of use of concrete under water;
- no counting method included;
- Miner number equal to 0.8.

VNC method for plain-concrete roads [16]

- load factor $\gamma_F = 1.0$ on unspecified load spectrum;
- material factor $\gamma_m = 1.0$ on characteristic flexural tensile strength after 90 days' curing;
- no fatigue limit;
- no influence of frequency;
- influence of wetness of concrete concealed, as can be concluded from discussion in [23];
- influence of stress gradient concealed in that the flexural tensile strength is assessed;
- no counting method included;
- Miner number equal to 0.5, being the mean value.

Draft Rules for Concrete Bridges, version of January 1986 [23]

- load factor $\gamma_F = 1.0$, owing service conditions being considered;
- material factor is not mentioned, because limits have been set to stresses;
- no thorough calculation of fatigue.

GTG 15 Fatigue of Concrete Structures [25]

- load factor is not mentioned explicitly; it will be specified in the Model Code of CEB;
- material factor $\gamma_m = 1.15$ for steel and $\gamma_m = 1.25$ for concrete;
- characteristic values for Wöhler curves, based on $\mu - 1.65\sigma$;
- Miner number equal to "1.0";

- the counting methods mentioned are the rainflow and the TNO method;
- no thorough calculation of fatigue.

It is concluded from this review that the various rules which include fatigue calculations still show serious gaps. The rules are inconsistent in their use of numerical values. Besides, the latter have no probabilistic basis. This appears for instance, from the range of Miner numbers used: 0.2 - 0.5 - 0.8 - 1.0. Values smaller than 1 suggest a safety margin with respect to the value of 1 mentioned by Miner. In fact, in the case of concrete the probability of collapse is virtually unaffected when the Miner number changes from 0.2 to 1.0. As regards VNC's value of 0.5 it should be remarked that, according to [22], this is the mean value, so that there is no margin in any case.
Among the various procedures, that of DNV gives a counting method. It is similar to that of TNO, but is not put on a scientific basis. GTG 15 refers to the rainflow and TNO counting methods.

3 Review of the literature

3.1 *General*

In the course of time several publications have appeared concerning design procedures for concrete structures subject to fatigue loading. In the present review, although limited in scope, all relevant aspects will be dealt with.
The symbols used in this chapter have been taken from the literature sources and are explained in the text.

3.2 *Fatigue Strength Evaluation of Offshore Concrete Structures [11]*

In this publication Waagaard gives the backgrounds of the design procedure formalized in the DNV Rules [8]. He compares them with other rules, such as those of the NPD, the FIP, the ACI and the TNO.
The FIP and the ACI rules are found to pay little, if any, attention to the fatigue aspect. It should be realized in this connection that the FIP rules considered by Waagaard were the predecessors of the FIP Recommendation [10] dealt with in chapter 2.
With regard to the fatigue procedure, the author comes to the following conclusions, briefly formulated:
- the random loading of offshore structures makes a fatigue analysis necessary;
- the Miner number involves uncertainties with respect to the material behaviour and the sequence of loading;
- a material factor has a greater influence on the attainable safety than a small change in the Miner number; this is due to the log-normal character of fatigue life;
- the use of Miner's Rule is rather time-consuming; therefore a rough estimate is needed to decide whether a detailed fatigue analysis is necessary.

3.3 *Analyse des effets des sollicitations répétitives sur les structures marines en béton [12]*

This publication of Zaleski-Zamenhof gives a critical comparison of rules concerning the analysis of concrete offshore structures subject to repeated loading, especially by the waves. She proposes a semi-probabilistic design procedure based on the following deterministic formulation:

$$\sum_{i=1}^{c} \frac{1}{N_i} \leq \frac{1}{k}$$

where

c = number of load cycles during service life
N_i = number of stress cycles giving fatigue failure in a constant-amplitude test at stress level i
k = safety factor by which the Miner number $M = 1/k$ is defined (see however, the remark in 2.3)

To adopt a semi-probabilistic design procedure, a relation of the following form has to be established:

$$S_d(F_c \cdot \gamma_f) \leq R_d(f_c/\gamma_m)$$

where

S_d = design value of the load effect
F_c = characteristic load
γ_f = load factor
R_d = design value of the load-bearing capacity
f_c = characteristic material strength
γ_m = material factor

The value S_d can be expressed as the design value of the Miner sum M_{Fd}:

$$M_{Fd} = \sum_{i=1}^{c} \frac{1}{N_i}$$

The load factor γ_F being taken into account to determine the stress level i.
The design value R_d can be expressed as a design value for the Miner number M_{fd}:

$$M_{fd} = \frac{M_{fc}}{\gamma_m}$$

Here M_{fc} is the characteristic value of the Miner number, which, according to [1] can be written as follows:

$$M_{fc} = 10^{-[\mu\,(\log M) - \beta \cdot \sigma\,(\log M)]}$$

where

$\mu(\log M)$ = the mean of log M
$\sigma(\log M)$ = the standard deviation of log M
β = reliability index

The material factor γ_m can be incorporated into the Miner number M_{fd} or in the Wöhler curve in the value of N_i. The above procedure was taken from [1 or 13].
The publication further discussed the fatigue procedures of the FIP of 1977, DNV of 1975 and of TNO-IBBC of 1975.
To evaluate the various design procedures, the design of a "Minipod" offshore structure has been worked out by means of the DNV and the TNO-IBBC procedure. The main conclusion from this exercise is that the TNO-IBBC procedure differs essentially from the DNV method in that it is based on a fatigue limit. Consequently, small fluctuations due not cause any fatigue damage. In the DNV procedure the greater part of the damage, or the Miner sum, originates from fluctuations below that "fatigue limit".
The DNV procedure works out at a Miner sum of 0.185, only little lower than the permissible value of 0.2 for the Miner number. According to the TNO-IBBC procedure these values are 0.002 and 0.8, respectively. In [14] Zaleski-Zamenhof presents a revised version of [12], in which the FIP recommendations of 1983 are discussed. In view of the definition of the design wave and the safety factors used in the ultimate limit state, it is conclused that for concrete offshore structure fatigue will play no role in the specification of dimensions.

3.4 *Ein Verfahren zur Berechnung der Betonfestigkeit unter schwellender Belastung [15]*

This publication describes a method for determining the fatigue strength of concrete under a certain number of load cycles. It introduces a parameter D, which describes the development of the microcrack in the concrete as a function of a critical stress. The influence of loading frequency is also considered.
Van Ornum et al. have determined the following non-linear relationship between the fatigue strength β_N and the number of stress cycles N.

$$\beta_{N_i}/\beta_c = C N_i^{-A}(1 + B \cdot R \cdot \log N_i) C_f$$

where

β_{N_i} = fatigue strength after N_i stress cycles
β_c = static compressive strength of the concrete
C = factor for the increased strength due to the increased rate of loading as compared with static test
C_f = factor for the loading frequency
A = factor for the critical stress
B = ditto
R = $\sigma'_{max}/\sigma'_{min}$

For factor C_f the following relation can be used:

$$C_f = 1 + a(1 - b \cdot R) \log f$$

where

a, b = constants, for instance $a = 0.07$ and $b = 1.00$
f = frequency of loading [Hz]

Constants a and b are chosen such that in the static determination of compressive cube strength the product $C_f \cdot b = 1$. Factors A and B are dependent on the critical stresses σ_I and σ_{II}. When the fatigue limit (or the fatigue strength at a very large number of stress cycles) is put equal to σ_I (e.g. $R = 0, f = 1$ Hz and $\log N = 9.5$) and if the long-term strength is put equal to σ_{II} (e.g. $R = 1$ and $\log N = 9.5$) then:

$$A = 0.008 - 0.118 \log (\sigma_I / \beta_c)$$

$$B = 0.118 \left(\frac{\sigma_{II}}{\sigma_I} - 1 \right)$$

The values of σ_I and σ_{II} can be determined, for example, by measuring the lateral and longitudinal deformations (including the change in volume) and hence determining the lateral contraction coefficient ν.
In the simplest case, where the fatigue strength β_{N_i} is dependent on the damage parameter D, the following relation is obtained:

$$\beta_{N_i} = k \cdot D + m$$

Parameter D is dependent on the change in lateral contraction ν with increasing load or number of stress cycles. Limiting conditions for the above relation are:

$$\beta_{N_i} = \beta_c \quad \text{for} \quad D = 0$$

$$\beta_{N_i} = \beta_N = \sigma'_{max} \quad \text{for} \quad D = 1 (N_i = N)$$

This gives:

$$\frac{\beta_{N_i}}{\beta_c} = (\beta c - \sigma_{max}) \cdot D$$

or:

$$\frac{\beta_{N_i}}{\beta_c} = 1 - (1 - \varkappa) \cdot D$$

The decrease in compressive strength of the concrete after N_i stress cycles is equal to:

$$\Delta \beta_c = (\beta_c - \sigma_{max}) \cdot D$$

or:

$$\frac{\Delta \beta_c}{\beta_c} = (1 - \varkappa) \cdot D$$

3.5 *Design of Concrete Structures for Fatigue Reliability [17]*

Four different formats for limit states with regard to the fatigue design of reinforced and prestressed concrete structures are compared in the light of a reliability analysis. These formats are concerned with:

- stresses;
- stress cycles;
- damage;
- static strength.

In the reliability analysis of structures the influence S^* of the load is compared with the capacity R^* [18]. In the various formats, S^* and R^* are expressed in stresses, stress cycles, damage and strength, respectively.

The stress format is based on a Wöhler curve, and makes full allowance for the distribution function around the curve. To simplify things, it is assumed that the minimum stress does not affect the fatigue strength. The maximum stress that occurs, σ_S, and the fatigue strength, f_R, are compared at the design service life N_S^*. A reliability function Z can be formulated as follows:

$$Z = f_R - \sigma_S$$

from which the probability of failure $P_f = P\{Z < 0\}$ or the reliability index β can be calculated, using the well-known techniques [18].

The procedure can also be set up as a semi-probabilistic analysis by defining the design fatigue strength f_R^* as:

$$f_R^* = \frac{1}{\gamma_{R\sigma}} \cdot f_{Rk}$$

where

$\gamma_{R\sigma}$ = the material factor
f_{Rk} = the characteristic fatigue strength

Similarly, for the design stress σ_S^* we have:

$$\sigma_S^* = \gamma_{S\sigma} \cdot \sigma_{Sk}$$

where

$\gamma_{S\sigma}$ = the load factor
σ_{Sk} = the characteristic fatigue strength

The design procedure can be summarized as follows:

$$f_R^* \geq \sigma_S^*$$

The stress cycles format is again based on a Wöhler curve. The capacity R^* is expressed as the number of cycles N_R^*, which the concrete can endure at the design stress σ_S^*. Since N_R is plotted on a logarithmic scale, the obvious definition of the reliability function Z is:

$$Z = \log N_R - \log N_S$$

where N_S is the number of load cycles that will occur during the design service life. From Z, in turn, the probability of failure P_f or the reliability index β can be calculated. (N.B. If the correct techniques are used, this would also be possible if the reliability function had been defined as $Z = N_R - N_S$).

The semi-probabilistic design procedure can be formulated as follows:

$$N_R^* = \frac{1}{\gamma_{RN}} \cdot N_{Rk}$$

where

N_R^* = the design value for the number of cycles to be endured
γ_{RN} = the material factor
N_{Rk} = the characteristic value of the number of cycles to be endured

Simularly, for the load element N_S we have:

$$N_S^* = \gamma_{SN} \cdot N_{Sk}$$

where

N_S^* = the design value for the number of cycles occurring
γ_{SN} = the load factor
N_{Sk} = the characteristic value of the number of cycles occurring; this value is to be deduced from the relevant distribution function $F(N_S)$

In this approach it is also possible to deduce the design stress σ_S^* from the characteristic value σ_{Sk} by introducing a partial safety factor γ_σ:

$$\sigma_S^* = \gamma_\sigma \cdot \sigma_{Sk}$$

If the design procedure is written in the damage format, it is assumed that each cycle causes a (possibly hypothetical) contribution ΔD to the damage. The value of ΔD can depend on various parameters, such as maximum stress, minimum stress, frequency, type of material, etc. The contributions ΔD added up to give a value D, which rises monotonically with the number of cycles. When a certain limit is exceeded fracture occurs. That limit can, in principle, be a deterministic or a stochastic value. In this case the reliability function Z is written as:

$$Z = D_R - D_S$$

where

D_R = the summated contributions to the damage caused by the load cycles during the design service life
D_S = the critical value of the damage at which failure occurs; this may be a deterministic value

By means of the distribution functions of D_R and D_S the probability of failure P_f and the reliability index β can be calculated. The semi-probabilistic calculation can be made using the design values D_S^* and D_R^*, for which the following relations hold:

$$D_S^* = \gamma_{SD} \cdot D_{SK}$$

$$D_R^* = \frac{1}{\gamma_{RD}} \cdot D_{RK}$$

where

γ_{SD} = the load factor
D_{SK} = the characteristic value of the summated contributions to damage
γ_{RD} = the material factor
D_{RK} = the characteristic value of the damage limit

In a number of cases fatigue failure occurs because the static load-carrying capacity has decreased as a result of fatigue cracking or other structural degradations. In such cases a format can be used which is based on the instantaneous static strength.
The reliability function Z can be defined as:

$$Z = S(t) - R(t)$$

It should be noted here that there may be a correlation because the strength R at a certain moment is dependent on the load exerted until that moment. This format differs from existing design procedures for static loads in that R and S are dependent on time, or on the number of cycles.
In principle, any of the four formats presented may be used. The ultimate choice depends on the materials employed, the section considered and the calculation methods used for other structural elements.
For the reliability analysis a target has still to be fixed for the reliability index β. Here essentially the same criteria apply as in the case of static loads. If the value of β is calibrated against that of statically loaded structures, the following proposal emerges:

$$2.75 < \beta\ \text{(fatigue)} < 4.0$$

Some examples are given of how partial safety factors are calculated according to the stress format and the cycle format.

3.6 *Probabilistic Reliability Analysis for the Fatigue Limit State of Gravity and Jacket-type Structures [19]*

For two types of offshore structures, namely a concrete gravity structure on four legs and a steel jacket structure also on four legs, a probabilistic reliability analysis is carried out. The wave loading has been modelled here as a stationary Gaussian process with a multi-directional Pierson-Moskowitz spectrum. For the long-term statistical parameters it is assumed that the significant wave height has a Weibul distribution and that the main direction of the waves is distributed uniformly.

The response of the structure has been calculated on the basis of a "model" analysis. To estimate the fatigue damage, use has been made of Miner's Rule for both the concrete and the steel structure. In the case of the concrete structure the fatigue analysis relates to the connection of one of the legs to the caisson. For the gravity structure a reliability index of $\beta = 4.8$ is found.
Analysis of the computational results shows that the calculation comprises a number of variables which make a relatively large contribution to the variance of the Z function.

These variable are:

C_M	= the inertia factor	25%
E	= modulus of elasticity	7%
ζ	= damping ratio	33%
s_F	= intersection in S-N curve	11%
D_F	= failure value of the Miner sum	12%
	total	88%

The first parameter is related to the description of the waves, the next two to the structural model and the last two to the fatigue model. Each of the basic elements of the calculation: load, structure and material behaviour, makes an essential contribution to the variance of Z. The relatively smallest contribution is that of the material behaviour. It should be pointed out here that in the fatigue model use has been made of the information obtained from the studies [1 to 5]. To simplify the calculation procedure a single Wöhler curve was used, the dependence from the minimum stress thus being ignored. The study does indicate, however, that a probabilistic approach to the problem is quite feasible.

3.7 *Probabilistic Analysis of Fatigue in a Concrete Offshore Structure [20]*

In this report on a student's final project a probabilistic analysis is made of the safety of an offshore gravity structure (referred to as ASTRID) as regards the fatigue aspect. The calculation is concentrated on a critical point in the upper part of the central shaft. This point is located some 30 metres underwater.
The study aims as a complete modelling of the waves of the structure itself and of the fatigue behaviour. In the present review only the fatigue model and the results will be discussed.
The fatigue model is based on Miner's Rule in combination with S-N lines (Wöhler curves). Miner's Rule is formulated as follows:

$$D = \sum_{i=1}^{j} \frac{n_i}{N_i} < D_f$$

$$N_i = \left[\frac{2\hat{s}}{S_f}\right]^{-k}$$

where

D = Miner sum
D_f = critical limit for the Miner sum
$2\hat{s}_i$ = range of stresses
k = constant dependent on material and stress level
S_f = ditto
n_i = number of stress cycles within range $2\hat{s}_i$
N_i = number of stress cycles within range $2\hat{s}_i$ to fracture

This model also involves a simplification of fatigue behaviour. The dependence on the mean stress is not explicitly considered. This aspect is made allowance for indirectly, however, in the values assigned to constants k and S_f.
In determining the shape of the Wöhler curve the author was faced with the problem that in the literature there is no unanimity on this point and that the range which is important for design purposes lies for the greater part outside the range of experimental studies (viz. up to 2,000,000 cycles). This is illustrated in Fig. 13. Eventually, a middle course was adopted. In principle, the reliability function Z is formulated as:

$$Z = D_f - D_{tot}$$

But in view of the logarithmic nature of Z and to avoid numerical inaccuracies, a slightly different formulation of the reliability function has been chosen:

$$Z = -\ln\left[\frac{D_{tot}}{D_f}\right]$$

Table 1 gives a review of the input data and the results of the probabilistic approach. The result of the calculation is found to be a reliability index $\beta = 4.0$. Major contributions to the variance of Z were found to come from:

C_m = diffraction factor, 15%
ξ_1 = damping of first Eigen frequency, 17%
S_f, k = fatigue parameter, 40%

The contribution of the fatigue parameters k and S_f to the variance of Z is about 40%. This large contribution can be explained from the logarithmic nature of the Wöhler curve, a small change in the slope of that curve leading to a considerable change in the damage (Miner sum). In the preceding study [19] the corresponding contribution was 11%. In that case, however, k was deterministic and the variation factor of S_f was only 0.10.
A striking aspect of the outcome of the probabilistic analysis is the very small contribution of the critical Miner sum D_f to the variance of Z, in spite of the fact that a variation factor of 0.25 was taken into account.
From the results of the probabilistic analysis partial safety factors for all the basic variables in the calculation have been derived. These are, in fact, central safety factors

Table 1. Probabilistic analysis data [20]

variable	description	distribution	μ	V	α^2
m_{dek}	payload	normal	60,000 ton	5 %	3 %
c_u	stiffness of soil	log-normal	70 kPa	25 %	8 %
E	modulus of elasticity	normal	40 GPa	5 %	2 %
t	wall thickness	normal	1.1 m	5 %	1 %
C_m	diffraction factor	log-normal	2.0	15 %	15 %
ξ_1	damping first Eigen frequency	log-normal	0.03	35 %	17 %
C_{1T}^*	fatigue parameter*	log-normal	136 MPa	20 %	40 %
k^*	fatigue parameter*	normal	7.5	10 %	
D_f	Miner sum	log-normal	1.5	15 %	1 %
d	water depth	normal	340 m	2 %	5 %
MO	additional inertias	log-normal	1.0	25 %	0 %
					$\Sigma\alpha^2 = 100$ %
					$\beta = 4.0$

* correlated variables

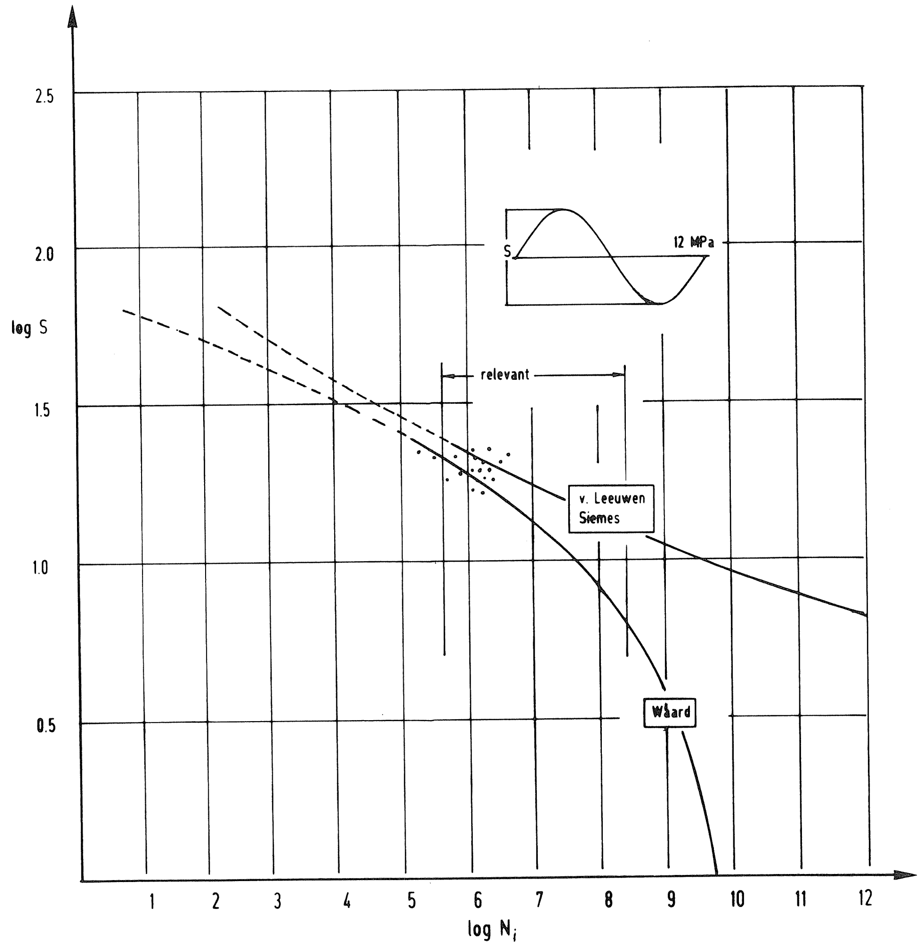


Fig. 13. Extrapolation of experimental Wöhler curves [20].

because they are related to the mean value of the basic variable concerned. The result was as follows:

payload	$\gamma_{sm\,dek} = 1.0$
additional inertia of foundation	$\gamma_{sc} = 1.0$
modulus of elasticity	$\gamma_{mE} = 1.0$
wall thickness	$\gamma_{mt} = 1.0$
C_m-value	$\gamma_{sCm} = 1.3$
damping	$\gamma_{s\xi} = 1.6$
critical Miner sum	$\gamma_{mDf} = 1.0$
distance between section and water level	$\gamma_{sA} = 1.2$
soil stiffness	$\gamma_{nCa} = 1.4$

Since in the calculation S_f and k have been assumed to be correlated, their contributions to the variance of Z cannot be separated and no partial safety factors can be determined. The design points were found to be:

$$S_f = 200 \text{ MPa}$$
$$k = 5.7$$

The mean values used were:

$$\mu(S_f) = 136 \text{ MPa}$$
$$\mu(k) = 7.5$$

If there should be no correlation, the respective central safety factors would be:

$$\gamma_{mSf} = 200/136 = 1.5$$
$$\gamma_{mk} = 7.5/5.7 = 1.3$$

3.8 *Concluding remarks*

In this chapter a review was given of a number of publications concerning procedures for the calculation of fatigue in concrete structures. The review does not pretend to be complete. In fact, literature on the subject is still scanty and that is not surprising. In connection with oil production activities on the North Sea a series of concrete offshore structures was built towards the end of the seventies. These activities called for design procedures that made allowance for the fatigue limit state. These procedures were based on the knowledge available at the time. Examples are [8] and [10]. Partly because there was felt to be a lack of knowledge, extensive experimental studies were then untertaken, the results of which have become generally available in the last few years. These findings have led to an adjustment of the design rules, such as [6] and [14] and to what may be expected to become a larger flow of publications.
On the other hand, it should be admitted that in those rules many aspects until then unknown were estimated with remarkable accuracy on the basis of good engineering judgement.

From the literature that has appeared so far a number of important facts emerge:
- in the present fatigue analysis procedures there are still a good many things that lack a firm basis [11]; examples are the numerical values to be used and the probabilistic interpretation of the parameters, the existence of a fatigue limit and the influence of specific conditions, such as the presence of water and the influence of loading frequency;
- on the whole, the existing procedures are based on different information (see chapter 2 and [11, 12, 14 and 20]; the differences concern, among other things, the shape of the Wöhler curves, the existence of a fatigue limit, the scatter in data and the value of the Miner number;
- by indirect routes it is attempted to derive a satisfactory value for the fatigue limit [15];
- by approaching the problem via reliability analysis it is attempted to lay a basis for the formulation of calculation procedures for the fatigue limit state; on the one hand this approach is in the form of somewhat theoretical foundations [17], on the other hand in the form of practical applications [19 and 20].

4 Design procedure

4.1 *Introduction*

In setting up a procedure for the assessment of the fatigue limit state it should be realized that fatigue will play a role in only certain types of structures. A major aim should therefore be to delimit the problem area by keeping the calculation procedure as simple as possible. A more or less obvious way of doing this is by looking at the maximum number of load cycles to be expected and/or at the largest stress cycle. If in a certain structure the fatigue aspect is found to need further attention there are, in principle, two possibilities:
- the calculations are carried out on the basis of fatigue properties indicated in the calculation procedure;
- the calculations are based on fatigue properties derived from targeted experimental research.

In the first option the material properties will have to be determined in a conservative manner, because there is no direct relation between these properties and the structure to be designed. There may be differences with respect to concrete composition, curing conditions or conditions of exposure.
If the properties are established via experimental studies, the procedure will have to indicate how the design properties should be derived from them. Another question to be considered in this case is to what extent and in what way the relations found may be extrapolated. Summarizing, the procedure will thus be built up as follows:
A. delimitation of the problem area;
B. procedure based on assumed properties;
C. procedure based on experimental results.

From the information presented in chapter 2 and 3 it may further be concluded that in the procedure the following aspects also need to be considered:
- the shape of the Wöhler curves and Goodman diagrams;
- the presence or absence of a fatigue limit;
- the log-normal character of fatigue life;
- the influence of such parameters as concrete grade, frequency of loading and location under or above water;
- Miner's Rule;
- the stress gradient;
- the interaction between concrete and steel;
- changes in E modulus;
- the principle of characteristic values and material factors.

Although the fatigue strength procedure cannot be isolated from the loading data and from the mechanical (dynamic) behaviour of the structures, attention will here be focussed mainly on the material. The other data will have to be supplied in due course by the disciplines concerned. In the procedure to be presented it has been tried, at least to indicate roughly how to handle load and mechanical data.

In [17] Warner argued that the calculation procedure can, in principle, be formulated in four different "formats".

a. stresses;

b. cycles;

c. damage;

d. static strength.

In view of the fact that the fluctuating loads on the structure under consideration are generally random in nature, method c would seem to be the most promising one. Methods a and b are in fact based on constant-amplitude stresses. Indirectly (by the implicit inclusion of a damage rule) a and b could also be adapted to random stresses. But this would make the procedure less straightforward.

Basically, method d could also be applied to random stresses, be it with rather complex formulas and likewise using implicit damage rules. The main drawback of this method is that it does not fit in with the behaviour of concrete. From the physical point of view, stress fluctuations monotonic decrease in the static strength of concrete, but they do in the reinforcing steel. The growth of a single local crack, will eventually cause fatigue failure. As soon as the crack is present, the static strength of the bar will have decreased. The use of method d for the reinforcing steel and of another method for the concrete is again not conductive to clarity. If possible, the same method should be employed and it would seem that method c is, in principle, suitable for this purpose. Since several investigations [1 to 5] have shown that Miner's Rule adequately describes the behaviour of concrete under all sorts of random stresses, it may be concluded that method c should then be formulated in conformity with Miner's Rule.

4.2 *Summary*

The design procedure for concrete structures subject to fatigue loading has been set up so as to ensure optimum compatibility with the procedures in the Dutch TGB (Regulations for the Calculation of Building Structures). Here and there definitions and symbols used in the fatigue procedure deviate from these principles. Thus, justice could be done both to the condition that structural safety should be ensured and to the fact that this is an assessment of service life. Just as in a conventional design procedure for the static behaviour, the probability of collapse during the design service life should be sufficiently low. It should be bore in mind, however, that the damage due to the fluctuating loads responsible for the fatigue builds up gradually during service life of the structure.

The calculation procedure is presented as a semi-probabilistic design procedure. This means that use has been made of characteristic values and partial safety factors for loads and material properties. To determine the number and the magnitude of the cycles in the case of fluctuation loads, a counting method is presented: The TNO counting method.

Limit states are used to evaluate the performance of the structure. The number of limit states to be considered has been restricted. After all, in addition to the fatigue analysis the static loads are evaluated as well.

The material properties to be used can be obtained in two ways. In the first place the procedure indicates values taken from recent research, but the properties may also be obtained from targeted experimental studies. In that case the experiments should simulate real-life conditions (type of concrete, curing conditions, test conditions, etc.) as closely as possible. The way in which such experiments should be carried out and interpreted is indicated.

The procedure used in judging whether fatigue will play a role in the structure comprises three successive steps:

- first it is checked whether the number of cycles exceeds a minimum;
- next it is established whether the magnitude of the stress cycles exceeds a certain limit;
- then, if necessary, a complete fatigue analysis is carried out on the basis of Miner's Rule.

In addition to the calculation procedure, recommendations are made with respect to design, execution, inspection and monitoring.

Fig. 14 gives a flow diagram of the whole procedure.

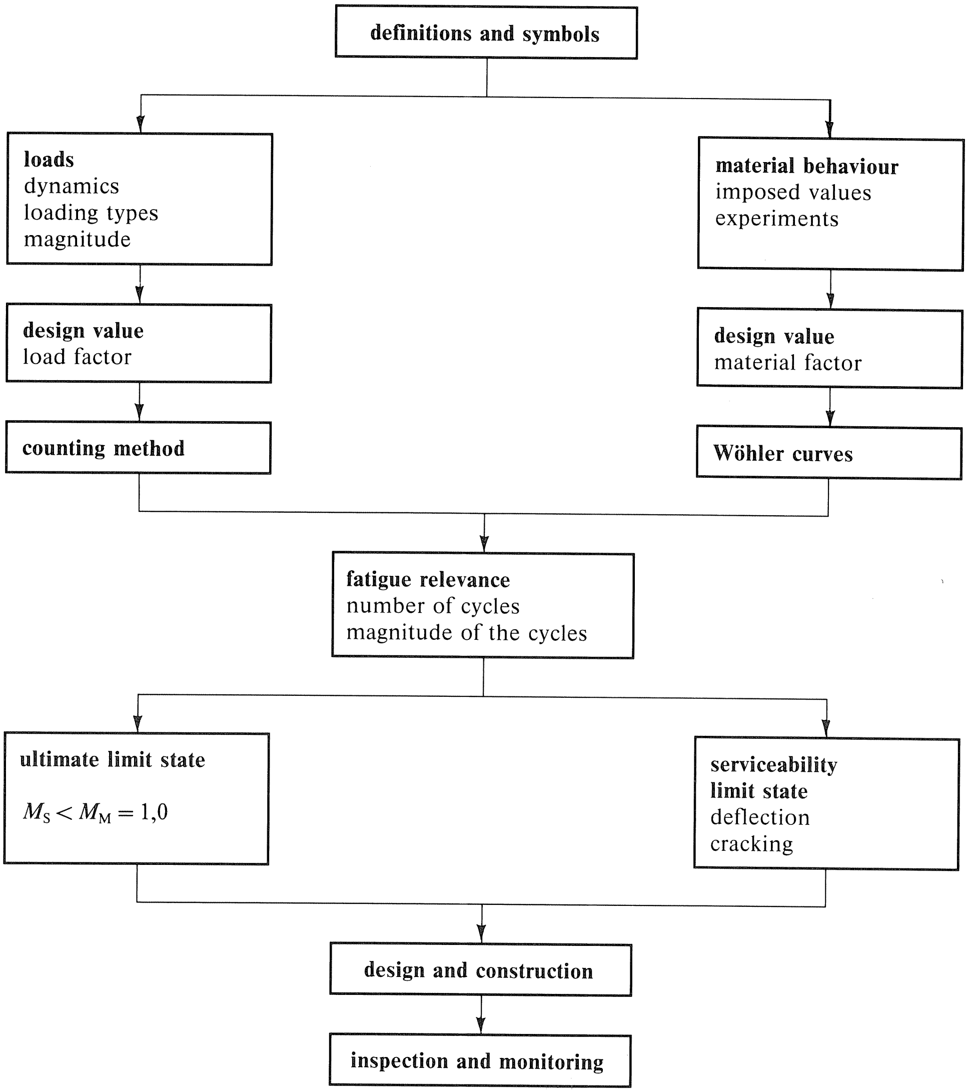


Fig. 14. Flow diagram of the assessment procedure.

Part II Fatigue strength procedure for concrete structures

In the calculation procedure to be presented the main text is in *italic* type. Explanations and remarks are in common type. In a later stage it will have to be decided which explanations and remarks should be maintained in the final version of this procedure. The literature referred to is listed after chapter 6 of part III.

1 Introduction

The study of fatigue in concrete structures subject to fluctuating loads is a still relatively new activity. A major incentive was the construction of large offshore structures for the production of oil and gas. In the design procedure proposed here it has been attempted to make optimum use of the present state of the art. But obviously more knowledge will be acquired in the course of time. Therefore, where possible, room was left in the procedure for improved data to be incorporated. One way of creating such room was by starting primarily from assumed material properties, yet leaving open the possibility of introducing per subject, other properties based on the outcome of experimental studies.

2 General

2.1 *Subject*

The present calculation procedure relates primarily to reinforced and/or prestressed concrete structures made from standard gravel concrete and in which part of the fluctuating load is clearly cyclic in nature. Examples in mind are offshore structures, chimneys, bridges, viaducts and roads.
With regard to the structures mentioned it is assumed that the design in respect of other relevant limit states has taken place or will take place in accordance with established rules for concrete.
The types of concrete structures mentioned under this heading include more than just offshore structures. It is felt that this need not be a drawback, fatigue being a general phenomenon. In the procedure it is assumed that other relevant limit states have been judged in conformity with the Dutch Rules for Concrete 1974/1984 [7]. This implies that the fatigue strength procedure need not be described in too much detail. A certain measure of reliability is already ensured by the static design procedure.
Limit states and loads have been dealt with mainly on the basis of the draft Dutch Standards NEN 6700 "Technical Principles for Structures, TGB 1986 - General basis" and NEN 6702 "Technical Principles for Structures, TGB 1986 - Loads".
If types of concrete other than gravel concrete, but containing no special additives, are used, the present evaluation procedure is, in principle applicable as well. The material properties should then be determined according to the method described in section 4.2.

In the above definition of the subject a restriction has been made with respect to the type of concrete. The restriction was dictated by the results obtained with special types of concrete, by CUR committee B30a, which has carried out some research on fatigue in the context of the MaTS studies [21]. The types of concrete tested had low water-cement ratios and high cement contents and contained silicafume and a light aggregate. The fatigue strength of the materials tested was found to be lower (relative to the static strength) than that of gravel concrete. Another important reason for this restriction is that relatively little is known yet about the relation between lightweight concrete properties and fatigue.
The construction of the concrete structures which forms the subject of the present fatigue strength procedure often involves special operations, such as the towing and installation of drilling platforms. It is assumed that professional skill will be exercised in making allowance for the design and execution.

2.2 *Definitions*

Reference period: *period of time within which the structure, during the various phases of its life, should continue to comply with the requirements laid down in this procedure. It is also the period to which loads and material properties refer.*
The sum of the reference periods forms the design service life. The actual service life may deviate considerably from this figure. In the various reference periods a distinction is made according to the various phases in the life of a structure, such as construction, transport, installation, commissioning, operation and break-up.

Loads: *any action that leads to stresses for deformations in the structure.*

Characteristic load: *value of the load linked with a certain probability of not being exceeded during the reference period.*
In the case of fluctuating loads the concept "characteristic value" is somewhat difficult to handle. For, in that case we are dealing with a process of relatively long duration (corresponding, in fact, with service life). The probability can then be linked with the process parameters, such as the mean value and/or the standard deviation.

Load factor: *safety factor which makes allowance for the uncertainty in assessing the load effect.*

Design load: *the value of the load to be adopted in the design procedure. It is obtained by multiplying the characteristic value by the load factor.*

Characteristic strength: *value of the strength linked with a certain probability of being the minimum attained in tests.*

Material factor: *safety factor which makes allowance for the uncertainty as to the material strength in the structure being attained.*

Design strength: *the value of the strength to be adopted in the design procedure. It is obtained by deviding the characteristic strength by the material factor.*

Reliability index *(β): measure of the nominal probability of a limit state being reached during the reference period.*

Limit state: *state in which one or several performance requirements to be met by the structure or parts thereof is (are) not complied with.*

Ultimate limit state: *limit state corresponding with the requirements as to maximum load-bearing capacity and the complete failure of the structural element concerned.*

Serviceability limit state: *limit state corresponding with the requirements for normal use.*

Fluctuating load: *a load varying frequently with time.*

Cycle: *change in load or stress occurring during a continuous period of time and consisting of a peak and a trough.*

2.3 *Symbols and dimensions*

β	*reliability index*
$\mu(X)$	*mean value of X*
$\mathrm{med}(X)$	*median value of X*
$\sigma(X)$	*standard deviation of X*
X	*characteristic value of X*
X_d	*design value of X*
f, f'	*tensile and compressive strength, respectively*
f'_{bkv}	*characteristic compressive strength of concrete subject to fatigue*
f'_{bkt}	*instantaneous characteristic compressive strength of concrete*
f'_{dv}	*design value of compressive strength of concrete for fatigue calculation*
f_{dv}	*design value of tensile strength of concrete for fatigue calculation*
f_{ak}	*characteristic tensile strength of reinforcing steel*
f_{da}	*design value of tensile strength of reinforcing steel*
f_{pk}	*characteristic tensile strength of prestressing steel*
f_{dp}	*design value of tensile strength of prestressing steel*
d	*fatigue parameter of steel*
$F(t)$	*time-dependent load*
$f(t)$	*time-dependent strength*
M_S	*Miner sum*
M_M	*Miner number*
γ_F	*load factor*
γ_f	*material factor*
γ_p	*load factor for permanent load in ultimate limit state*
γ_r	*load factor for static load in ultimate limit state*

γ_{w}	*load factor for fluctuating load in ultimate limit state*
γ_{wd}	*load factor for fluctuating load with dynamic effects in ultimate limit state*
γ_{bp}	*load factor for permanent load in serviceability limit state*
γ_{br}	*load factor for static load in serviceability limit state*
γ_{bw}	*load factor for fluctuating load in serviceability in limit state*
γ_{b}	*load factor for serviceability state*
ξ	*stress concentration factor for reinforcing steel*
Φ	*diameter of reinforcing bar*
$\sigma(t)$	*time-dependent stress*
σ_{max}	*maximum stress in a cycle*
σ_{min}	*minimum stress in a cycle*
R	*stress ratio* $\sigma_{min}/\sigma_{max}$
ω	*cycle frequency*
T	*period* $T = 1/\omega$
σ/f	*relative stress*

Dimensions
All units should conform to the SI system of units.

2.4 *Summary of design procedure*

First a review is given of a number of terms used in this procedure together with their definitions. They conform to the (Dutch) TGB standard and to the semi-probabilistic design procedure used in that standard.
Next, the various types of loading that may play a role are discussed. In particular, it is argued that fatigue analysis is a combined assessment of safety and of service life.
Material behaviour is included in the procedure in two different ways: in the first place through assumed properties and secondly through properties that can be determined experimentally. In the latter case the test procedure is indicated.
The operation of the structure is judged by considering limit states. Both serviceability and ultimate limit states are considered. The fatigue analysis can then be performed at three different levels. At the first level it is established whether the number of load cycles exceeds a certain minimum. At the second level the magnitude of the stresses is examined. At the third level an extensive analysis is performed on the basis of Miner's Rule. Finally, some general instructions are given concerning the design and execution and recommendations are made as to inspection and repairs.

3 *Loads and dynamic behaviour*

3.1 *General*

In the assessment of the fatigue aspect all relevant loads that may occur during the successive reference periods should be taken into account.

Loads are considered to be relevant when the probability of their occurrence during the reference period is greater than about 50 %. Exceptional loads, by virtue of their nature, are not included. Examples of such occasional events are collisions, crashes, explosions, earthquakes in this region and the like.
In view of the fact that for the concrete structures under discussion special techniques are used, allowance should be made for the effects of special loads during the execution, such as towing forces, installation forces, and the like.

3.2 *Types of load*

A load comprises:

- *direct loads, i.e. one or more concentrated or distributed forces acting on the structure;*
- *indirect loads, i.e. deformations imposed on the structure.*

Thermal stresses are an example of indirect loads. If present, they should be taken into account in the assessment of fatigue.

For the purpose of the present procedure, loads are divided into:

- *permanent loads; these vary only little in magnitude during the reference period;*
- *variable loads; these are not constant throughout the reference period, or the variations as a function of time are not negligible;*
- *exceptional loads; these have a considerable effect on the structure, but the probability of their occurrence during the reference period is low.*

The variable loads are distinguished into:

- *static loads; these change a limited number of times during the reference period; in the present procedure all variable loads persisting for more than one minute are considered to be static;*
- *fluctuating loads; these change a great many times during the reference period; in the present procedure all variable loads are regarded to belong to this category if a load cycle takes less than one minute.*

The criterion of one minute for the distinction between static and fluctuating loads is arbitrary. The ultimate results of the fatigue assessment would hardly be affected if a criterion of one hour was chosen.
As a matter of fact, a fluctuating load in the present context means that fatigue effects cannot be ruled out. But this fact is not firmly established until the whole procedure has been completed.
In the fatigue assessment exceptional loads may be left out of consideration.

3.3 *Determination of loads and effects*

The magnitude of the loads should be determined for the reference period considered. Of the fluctuating loads the type of distribution and the parameters of that distribution have to be determined. These data should comprise at least the maximum and the minimum value, as well as the frequency of the load cycles. In calculating the effects of the fluctuating loads, the influence of permanent and static loads should also be taken into account.

Fluctuating loads should be determined in the same way as the corresponding loads considered in the static analysis.
In principle, (semi-)static calculations on the ultimate limit state will also include fluctuating loads. Thus, from the wave load a design wave is deduced which is defined probabilistically. It may be advantageous from the computational point of view to carry out the fatigue analysis on the basis of a distribution of all the instantaneous wave tops, and to assess the ultimate limit state on the distribution of the basis of the waves over a certain period (for instance, a gale or a year). This approach should not lead, however, to the two distributions being in conflict with each other.
Non-stationary fluctuating loads should be taken into account as a load programme consisting of several periods of stationary loads. The number of periods to be taken into account should be chosen with due regards to the nature of the load being considered. It should be demonstrated that the division into periods leads to a conservative approach in the assessment of the fatigue effect.
From the CUR/MaTS studies [5] it has become clear that non-stationary stresses may be taken account of the same way as stationary ones. To avoid becoming entangled in unnecessarily complicated procedures, the non-stationary signal can be conceived as consisting of several periods within which the signal is more or less stationary.
Periods of rest may be neglected.
Previous studies have shown that periods of rest may be completely disregarded.
The computational models used in calculating load effects should give a reliable picture of the actual behaviour of the structure. In the assessment of fatigue effects the theory of linear elasticity generally give a sufficiently reliable picture.
Insofar as fluctuating loads cause dynamic effects, these should be included in the calculation of the load effects.
The construction of large offshore structures has entailed an improved insight into the dynamic behaviour of structures. A better insight has also been obtained into the determination of forces due to wind, waves, tides, etc. The calculation methods to be used are often subject to strict limiting conditions. Thus, spectral calculations are permissible only when there is sufficient linearity between the source of the load and the resultant force exerted on the structure.
The principle adopted in dividing the loads into various types has made it possible for the fatigue analysis to be carried out using load periods in which per period the influence of the following factors is taken into account:
- the permanent load, which only influences the level of the mean stress;
- the static load, insofar as it is included in the block concerned and which likewise only influences the level of the mean stress;
- the fluctuating load, which has been split up into blocks of loads of a stationary nature; this load may influence the level of the mean stress.

In cases where several load periods have to be considered, simplifications are possible, making allowance for a possible correlation between the various loads.
If any simplifications are introduced, these should be demonstrated to be justified. The

approximation should then lead to a higher degree of damage than the more accurate calculation. Possibilities of simplification are offered, for instance, by:
- the dynamic behaviour of the structure;
- anticipating application of the TNO counting method.

The calculations to be carried out should be based on characteristic values for the loads. During the reference period the level of significance of the characteristic values should not exceed the following percentages:
- *permanent load 5 %*
- *static load 5 %*
- *fluctuating load 50 %*

The percentage of 50 % for the fluctuating load relates to the parameters (mean and standard deviation) of the distribution. It is in agreement with current practice in measuring loads. It is possible in principle, by means of recording, to obtain an impression of the mean and the standard deviation of the load. But as a rule it is not feasible to establish the reliability of the parameters.

The duration of the various types of load should be related to a level of significance of 50 %. For combinations of loads no reduction factors should be applied.

The techniques adopted in the present procedure is such that the (im)probability of a combination of loads occurring has been made allowance for in the probability of the characteristic load to be taken into account. The approach conforms to the Turkstra rule, which states that in the case of load combinations one load should be assumed to be the extreme one, the other being instantaneous.

If necessary, use will have to be made of the loads stated in specific rules.

An example is the Draft Rules for Concrete Bridges (in Dutch).

For the ultimate limit states the load factors to be taken into account are:
- *permanent load* γ_p
- *static load* γ_r
- *fluctuating load* γ_w

Further probabilistic analysis of various limit states should lead to a final specification of the values.

For the time being, it is proposed to use the following values for the load factors:

$$\gamma_p = 1.0$$
$$\gamma_r = 1.2$$
$$\gamma_w = 1.3$$

As far as the prestressing force is concerned, the obvious value to be chosen for γ_p is 1.0, because otherwise the interplay of forces in the section would become different.

If the permanent load has a favourable effect on the load-bearing capacity, a load factor equal to 0.9 should be adopted.

For the fluctuating load a value equal to γ_{wd} *should be adopted if dynamic effects play a major role.*

The studies reported in [19] and [20] have shown that dynamic calculations, wave force calculations and the like are still essentially unreliable. It is felt that allowance should

be made for this uncertainty by increasing the load factor for the fluctuating load. For the time being a value of 1.4 is recommended for γ_{wd}.
The load factors proposed here are lower than those specified in the draft Dutch Standard NEN 6702 "Regulations for the Calculation of Building Structures, 1986 - Loading". It is believed, however, that the uncertainties are sufficiently taken account of by characteristic values valid for the whole reference period, but this opinion has still to be verified by further calculations.
For the serviceability limit states the following load factors have to be taken into account:
- *permanent load* γ_{pb}
- *static load* γ_{br}
- *fluctuating load* γ_{bw}

Here, too, it is felt that sufficient allowance is made for uncertainties by the characteristic values of the loads. This applies also to the fluctuating load. The greater uncertainty in that load is cancelled out by the fact that the effects relating to the use are compensated for by accurate assessments of fatigue in the ultimate limit state. For the time being, values of $\gamma_b = 1.0$ are recommended for the partial load factors.

4 *Material behaviour*

4.1 *Assumed material properties*

The material properties not mentioned in the present procedure may be taken directly from existing specifications.
Among the properties not mentioned in these rules are the modulus of elasticity, strain at break and the like. In general these properties are to be found in the "Technical Principles for Structures - Concrete" (in Dutch).
In the case of reinforced or prestressed concrete structures allowance should be made for the possibility of fatigue in at least the following materials and stress conditions:
- *concrete under compression;*
- *concrete under tension or shear;*
- *bond;*
- *reinforcing steel;*
- *prestressing steel;*
- *prestressing anchors.*

In view of the different behaviour of the materials mentioned and of the different load combinations considered, allowance should be made for the possibility of other types of limit states than those occurring in static calculations becoming critical.
The present procedure applies in principle to concrete of grade B 37.5. The effect of higher concrete grades may be taken into account. The higher concrete grade may be the result of:
- *a better concrete mix composition;*
- *improved curing conditions;*
- *prolonged curing.*

The characteristic compressive strength f'_{bkv} *of the concrete to be taken into account in assessing fatigue can be derived from the instantaneous characteristic compressive strength* *of the concrete, f'_{bkt}, and for the actual curing conditions of the concrete in the structure. This conversion takes place as folllows:*

$$f'_{bkv} = \left(\frac{f'_{bkt} - 30}{2}\right) + 30$$

where

f'_{bkv} = *characteristic compressive strength of the concrete to be taken into account for fatigue*

f'_{bkt} = *characteristic compressive strength of the concrete after t day's curing and under curing conditions t*

For concrete grades lower than B 30:

$$f'_{bkv} = f'_{bkt}$$

For the material factor the following value should be taken:

$$\gamma_m = 1.25$$

From the value f'_{bkv} the design value of the compressive strength for use in fatigue calculations, f'_{dv}, can be derived. These design values are:

a. for flexure or flexure with a small normal compressive force:

$$f'_{dv} = \frac{f'_{bkv}}{\gamma_m} = 0.8 f'_{bkv}$$

b. for flexure with a large normal compressive force:

$$f'_{dv} = \frac{0.85 f'_{bkv}}{\gamma_m} = 0.7 f'_{bkv}$$

c. for flexure with tensile force:

$$f'_{dv} = \frac{f'_{bkv}}{\gamma_m} = 0.8 f'_{bkv}$$

The values presented are in agreement with those of the Dutch Standard NEN 3880 "Rules for Concrete 1974/1984", except for the fact that a material factor has been taken into account and that the factor of 0.9 used to take into account the effect of long-term loading has, of course, been omitted.
The CUR/MaTS studies [5] have shown that eccentric compressive forces, such as those occurring in a compression zone loaded in flexure, can be calculated in the same way with static and with fluctuating loads. The relevant design values are based on this consideration.

The design value for the tensile strength of concrete subject to a fluctuating load can be calculated from the relation:

$$\boxed{f_{dv} = 0.6 + f'_{dv}/25}$$

where

f_{dv} = *the design value of the tensile strength for use in fatigue calculation, expressed in N/mm²*

The above relation is based on table A-15 of Dutch Standard NEN 3880 "Rules for Concrete 1974/1984", with the exception that the factor of 0.7 specified there in connection with the effects of the long-term loading has been disregarded. However, a somewhat higher factor has been taken into account because the formula for the calculation of the tensile strength contains a greater uncertainty.
For stress cycles with a frequency in the order of magnitude of 1 Hz the number of stress cycles N_i^1 which the concrete can endure under compression, can be derived from the following relation:

$$N_i^1 \text{ is unlimited if } \sigma_{max}/f'_{dv} < 0.25$$

$$\log N_i^1 = \frac{10}{\sqrt{(1-R)}}(1 - \sigma'_{max}/f'_{dv}) \quad \text{if} \quad \sigma'_{max}/f'_{dv} > 0.25$$

where

N_i^1 = *the number of cycles to fatigue failure at a frequency of 1 Hz*
R = *stress ratio in a cycle:* $R = \sigma'_{min\,i}/\sigma'_{max\,i}$

The above relation is represented graphically in Fig. 15.
The above relation is based on a fatigue limit corresponding with a value equal to $0.25f'_{dv}$. The existence or non-existence of a fatigue limit has never been convincingly proved by experimental evidence. The work reported in [13], however, has shown that, if a fatigue limit does exist, it will be below the value of $0.4f'_{dv}$. In the present procedure the value of $0.25f'_{dv}$ is used as the fatigue limit because under static loads up to this level no perceptible micro-cracking occurs.
It should be noted, however, that the introduction of a fatigue limit at a value of $0.25f'_{dv}$ does not materially affect the ultimate result of the calculation. Thus, at $R = 0$ it is then found that $N_i^1 > 10^{7.5}$. From the physical point of view, at a frequency of 1 Hz approximately $10^{9.2}$ cycles are possible in a normal reference period of 50 years:
- generally speaking, the fluctuating load will not be exerted continuously throughout the reference period (periods of rest!);
- for most loads the frequency of 1 Hz is on the high side;
- the assumption that $R = 0$ is usually unfavourable.

Consequently, for the outcome of the fatigue assessment it would make relatively little difference if no fatigue limit had been introduced. The relation for the Wöhler curve

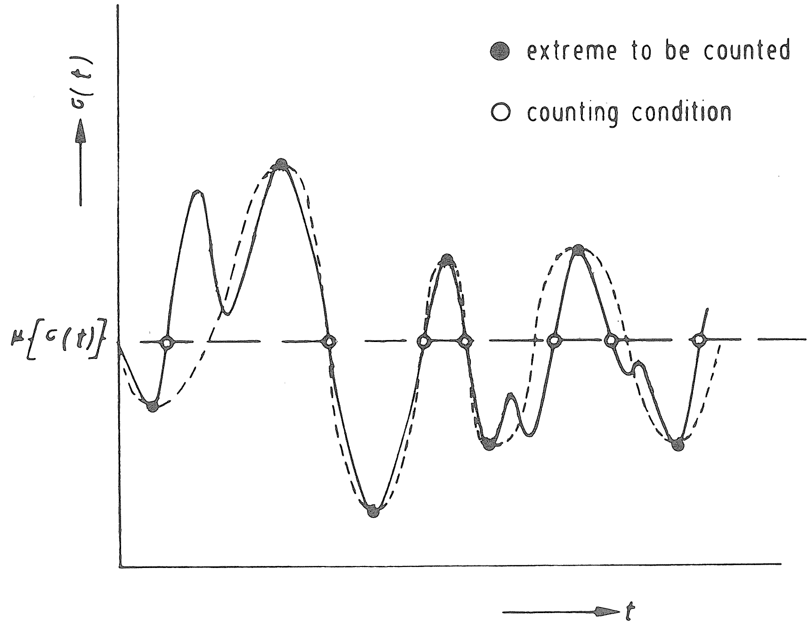


Fig. 15. Principle of TNO counting method.

has been formulated such that for $R = 1.0$ (or a static long-term load) the compressive strength of the concrete is equal to f'_{dv}. Although the value of this quantity does not match that of the long-term strength, there may be said to be a reasonable agreement.
For stress cycles with a frequency of 1 Hz the number of stress cycles N_i^1 that can be endured by the concrete under tensile stresses, can be derived from the following relation:

$$N_i^1 = \textit{unlimited if } \sigma_{max} < 0.25 f_{dv}$$
$$\log N_i^1 = 15(1 - \sigma_{max}/f_{dv})$$

The above relation was taken from [16]. For reasons of simplicity the slightly favourable effect of the minimum stress has been neglected.
For stress cycles with a frequency in the order of the magnitude of 1 Hz the number of stress cycles N_i^1 that can be endured by the concrete when subjected to stress alternating between tension and compression, can be derived from:

$$N_i^1 = \textit{unlimited if } \sigma_{max} < 0.25 f_{dv}$$
$$\log N_i^1 = 10(1 - \sigma_{max}/f_{dv})$$

From studies reported in [3] it emerged that stress cycles alternating between tension and compression cause extra fatigue damage. This effect is expressed in the above relation.

The effect of frequencies in the stress cycles deviating from the value of 1 Hz should, both for compressive and for tensile stresses in concrete, be taken into account by means of the following relation:

$$\log N_i^f = \log N_i^1 - 0.65 \log \frac{1}{f}$$

where

N_i^f = *the number of cycles to fatigue failure at a frequency of f Hz*
f = *frequency of a stress cycle*

If the frequency of the stress cycles is greater than 1 Hz, the frequency correction may be omitted.

The work reported in [5] has demonstrated that the frequency of the stress cycles has a certain influence on the number of stress cycles to fatigue failure. The correction for the frequency given above is a simplified form of the corresponding relation given in [5]. This simplification is justified by the relatively small influence of the frequency. In [4] it is stated that with tensile stresses the influence of frequency is similar to that under compressive stresses.

As far as the bond is concerned, no special measures need to be taken.

Relatively little is known yet about the behaviour of the bond zone under a fluctuating load. It does not seem sensible, therefore, to make any special demands in this respect. Since a static calculation is made as well, it may be expected that in most cases a sufficient anchoring or overlap will be achieved. Moreover, the fatigue assessment of the reinforcing bars leads to a reduction of the bar forces and hence to a decrease of the anchoring or overlap forces.

Furthermore, the conditions under which the present fatigue procedure may be employed are such that there may be assumed to be a complete interaction between the concrete and the steel in a section of the concrete, and hence an adequate bond.

The design values for the tensile strength of reinforcing steel and prestressing steel can be deduced from the respective characteristic strengths, taking into account a material factor:

$$\gamma_s = 1.0$$

This leads to

$$f_{da} = f_{ak}/\gamma_s = f_{ak}$$

and

$$f_{dp} = f_{pk}/\gamma_s = f_{pk}$$

where

q_s = *material factor for the reinforcing steel*
f_{ak} = *characteristic tensile strength of the reinforcing steel*
f_{da} = *design value of the tensile strength of the reinforcing steel*
f_{pk} = *characteristic value for the tensile strength of the prestressing steel*
f_{dp} = *design value of the tensile strength of the prestressing steel*

The number of stress cycles N_i *that can be endured by straight bars of reinforcing steel can be deduced from:*

$$\log N_i = \frac{6(1 - \sigma_{max})}{f_{da} - d - \sigma_{min}(1 - d)/f_{da}}$$

where

σ_{min} = *the minimum stress in the steel*
σ_{max} = *the maximum stress in the steel*
d = *fatigue parameter of the steel*

The values of the fatigue parameters of reinforcing steel are:

$d = 220\ N/mm^2$ *for FeB 500, HW*
$d = 190\ N/mm^2$ *for FeB 500, HK*

In connection witht the occurrence of adverse stress concentrations values d should be multiplied by a stress concentration factor ξ*, which is equal to:*

$\xi = 0.55$ *in the case of hooks with ratio of curvature* $> 10\Phi_k$
$\xi = 0.75$ *in the case of flash butt welding*
$\xi = 0.70$ *in the case of cross welding*

The values given for ξ have been taken from [23] and need further verification.
The fatigue properties of prestressing steel and prestressing anchors should be taken from the manufacturer's specifications. These relevant tests should have been carried out in conformity with the results laid down in 4.2.

4.2 *Properties derived from experimental research*

The fatigue properties of concrete, reinforcing steel and prestressing steel may, in principle, be derived from experimental studies.
The fatigue properties given in 4.1 give a simplified picture of the actual properties. This means, among other things, that the properties given are on the conservative side. Experimental evidence may narrow the gap.

In the tests to be carried out the experimental parameters should simulate the field conditions of the concrete structure to be designed, as closely as possible.
If in the tests to be carried out with the concrete the experimental parameters are not in agreement with field conditions, a translation is possible with respect to:

- *concrete grade;*
- *degree of curing;*
- *curing and testing conditions;*
- *load frequency.*

This should be done by analogy with what was stated in 4.1.
In fatigue research the usual care should be exercised in the tests to be carried out. An aspect that calles for special care is the scatter.
This implies that each test should be repeated so often that the mean test result has a confidence limit of more than 95%.
In various experimental studies it has been found that fatigue tests show a considerable scatter in service life. Therefore, if the cost and the effort involved in experimental studies are to be worthwhile, a good deal of accuracy will be required in carrying out tests.
The definition of design values etc. given in 4.1 cannot be invalidated by experimental results.
If the outcome of experimental studies should be a reason to reconsider safety with respect to fatigue failure, this should be done through a level II or level III reliability analysis as described in 5.1.

5 *Limit states*

5.1 *Safety and serviceability requirements*

Concrete structures subject to fluctuating loads, should, during the reference period, meet the requirements laid down with respect to safety and serviceability, the measure of reliability being fixed in advance.
The requirements laid down with respect to safety and serviceability should be complied with throughout the reference period. This has implications for durability as well. Basically, the phenomenon of fatigue has resemblence with "durability", the cause of deterioration being mechanical, and hence fits in with the above general requirement (which is formulated in conformity with article 5.1.1 of the draft "Regulations for the Calculation of Building Structures, Concrete 1986 ('TGB Concrete' – General Basis, draft Dutch Standard NEN 6700"). For the sake of completeness, it is pointed out that fatigue may affect both the serviceability and the safety of a concrete structure. For instance, deformation or crack formation may increase and ultimately lead to failure. In older publications fatigue was considered only within the framework of the serviceability limit state. This was because, wrongly, a load factor γ_F of 1.0 was used.
In determining the reference period allowance should be made for the engineering conditions. Economic considerations may also play a role.

The reference period of a concrete structure under construction is equal to the time needed for execution and installation; for a complete concrete structure it is equal to the design service life (the planned period of use).
Unless other periods have been agreed upon, the reference period for the execution and installation, if any, is 3 years and for service 50 years.
The reliability of a concrete structure subject to fluctuating loads is expressed as the probability of a limit state being exceeded within the reference period.
The reliability of the structure may be determined by means of a reliability analysis at level II or III, but it may be assumed that, if the method given in the present procedure is followed at level I, the required reliability has been attained.
The Joint Committee on Structural Safety has defined four levels for the reliability analysis using probabilistic calculations. They vary from completely deterministic to completely probabilistic. These levels are the following:

Level 0: A deterministic calculation. For the loads and the load-bearing capacity certain fixed values are taken and the computational model is considered to be definite. All uncertainties are taken into account by one overall safety factor.

Level I: A semi-probabilistic calculation. Characteristic values are established for the loads and the load-bearing capacity. The remaining uncertainties are taken into account by means of partial safety factors, i.e. safety factors relating to individual quantities.

Level II: A probabilistic calculation in which well defined simplifications have been introduced in the handling of stochastic quantities. This can be done by different methods, such as the mean-value or the first-order second-moment method.

Level III: A complete probabilistic calculation. The calculation is based completely on stochastic theory.

The concrete structure and its component parts should have at least the following reliability index β:
- *ultimate limit state: 3.6*
- *serviceability limit state: 1.8*

In special cases another reliability index may be agreed upon.
Before a structure is judged with respect to fatigue the (quasi-) static load-bearing capacity should be sufficiently ensured. This condition is fulfilled when:
- *the calculation of the ultimate limit state according to "TGB-Concrete" has demonstrated that there is a sufficiently reliable load-bearing capacity;*
- *there is sufficient certainty that the structure is capable of bearing the following combination of loads:*
  - *dead weight and permanent load*
  - *static load*
  - *the largest stress cycle during the reference period.*

These calculations should be carried out using the same partial safety factors as in the assessment of the ultimate limit states.

The object of the above-mentioned calculation of the static ultimate limit state is to restrict to some extent the influence of the fluctuating load. This is necessary because it is still uncertain whether under all circumstances all components in a concrete section may be expected to show complete interaction [26 and 27].

The design values for the stresses should be determined from the effects of the characteristic loads multiplied by the appropriate load factors. This should be done in due allowance for the dynamic behaviour of the structures.

5.2 *Counting method*

The fluctuating stress to be taken into account in the assessment of fatigue (together with the contributions of the permanent and the static stress) should be simplified by means of the TNO counting method.

In the case of non-stationary random loads the counting method should be applied to periods which are more or less stationary.

The changes in stress as a function of time which are the outcome of the calculation are erratic in nature when a stress with a wide range of frequencies is concerned. Stresses with a narrow band of frequencies are more or less sinusoidal in relation to time. The investigations [5] carried out under the auspices of CUR/MaTS have shown that for a wide-range signal it is not necessary, even undesirable, to consider all the details of the signal in the assessment of fatigue. The signal should be reduced by means of the TNO counting method.

The TNO couting method comprises the following steps:

1. *Non-stationary random stresses are split up into periods of more or less stationary stresses;*
2. *Per period, the following calculations are performed:*
   - *the mean stress* $\mu[\sigma(t)]$ *is determined*
   - *the first passage through* $\mu[\sigma(t)]$ *is determined:* t_1
   - *the next two passages,* t_2, t_3, *are determined*
   - *the extreme value* $Extr\,[\sigma(t_1 \rightarrow t_2)]$ *of* $\sigma(t)$ *between* t_1 *and* t_2 *and its moment of occurence* t_1^* *is determined and* $Extr\,[\sigma(t_2 \rightarrow t_3)]$ *between* t_2 *and* t_3*and its moment of occurence* t_2^*
   - *the stress period* $\sigma(t_1 \rightarrow t_3)$ *is replaced by half a cosinusoidal cycle with the following characteristics:*
     - *double amplitude* $2A = Extr\,(\sigma_{t1} \rightarrow \sigma_{t2}) + Extr\,(\sigma_{t2} \rightarrow \sigma_{t3})$
     - *mean* $m(\sigma) = \left| \dfrac{Extr\,(\sigma_{t1} \rightarrow \sigma_{t2}) - Extr\,(\sigma_{t2} \rightarrow \sigma_{t3})}{2} \right|$
     - *frequency* $\omega = \dfrac{1}{2\,|\,t_2^* - t_1^*\,|}$
   - *stress period* $\sigma(t_1 \rightarrow t_2)$ *is replaced by half a cosinusoidal cycle with the following characteristics:*
     - *mean value equal to* $m[\sigma(t)]$
     - *amplitude equal to* A

- *frequency equal to ω*
- *the half cycle is rising or descending in accordance with the original stress*

- *the procedure is repeated for the subsequent passage of $\sigma(t)$ through the value $\mu[\sigma(t)]$, namely t_2 and t_3, etc.*
- *for the final assessment of fatigue the half cycles may be placed in any order desired.*

The procedure is represented graphically in Fig. A.

In this procedure only half cycles are determined. This is permissible as long as the rest of the procedure is likewise based on half cycles. Owing to the fact, demonstrated in [5], that the sequence of loading does not materially affect the outcome, the individual (half) cycles can be placed in an order that will facilitate the calculation procedure for example in blocks or in order of magnitude.

5.3 *Examining relevance to fatigue*

5.3.1 *General*

The fatigue limit state need not be considered when the number of stress cycles during the reference period is less than 1000.
Research has shown that a few hundred stress cycles at a high level are not enough to cause fatigue failure. Some extrapolation of that number seems justified because it may be assumed that in that case the static design procedure makes allowance for the possibility of fatigue failure. In the light of actual practice, 1000 is a very low figure. In a

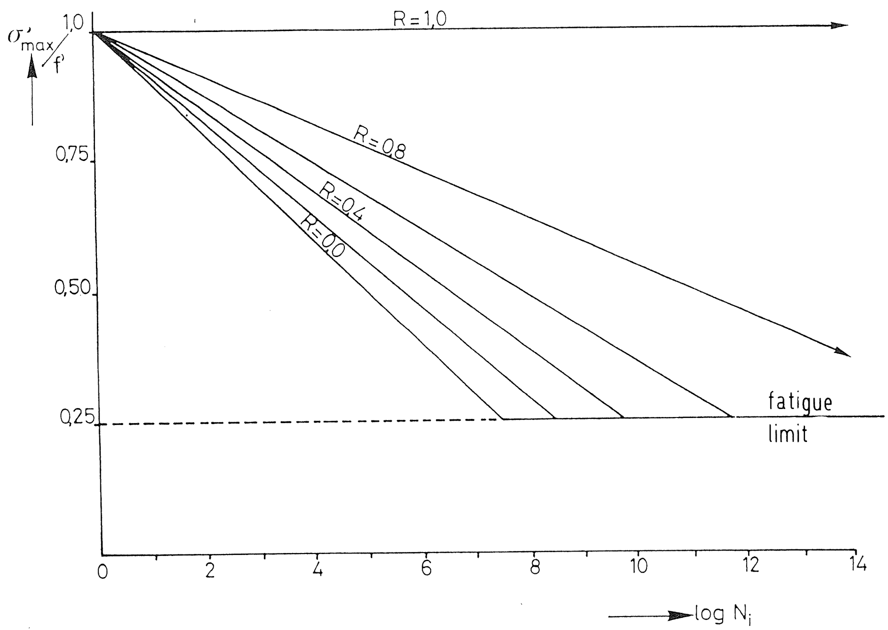


Fig. 16. Draft Wöhler diagram.

50-year period the number of temperature changes due to the night-and-day cycle alone is about 17,500.
With respect to the serviceability limit state as far as crack formation and deflection are concerned, it may be remarked that in the studies that have led to the calculation models the numbers of cycles were assumed to be of this order of magnitude.

5.3.2 *Compressive stresses in the concrete*

The highest maximum compressive stress occurring in the concrete during a stress cycle as a result of both the permanent and the fluctuating load, should obey any of the following relations:

$$\sigma'_{b\,max} \leq 0.25 f'_d$$

or

$$\sigma'_{b\,max} \leq f'_{dv}\left(1 - \frac{\log N}{12}\right)$$

$$\sigma'_{b\,max} \leq 1 - \frac{\sqrt{(1-R_i)}}{10} \log N_i$$

here

$\sigma'_{b\,max}$ = *the maximum compressive stress in the fibre of concrete considered*
f'_{dv} = *the design value of the compressive strength of the concrete for fatigue calculation*
R_i = *the ratio of* $\sigma'_{b\,max}/\sigma'_{b\,min}$
$\sigma'_{b\,min}$ = *the minimum compressive stress in the fibre of concrete considered; if at that location tensile stresses occur or crack formation has taken place,* $\sigma'_{b\,min}$ *may be taken to be equal to 0*
n = *number of stress cycles in the reference period(s)*

Fig. 17 is a Wöhler diagram showing the relation between maximum compressive stress, minimum compressive stress and number of cycles from which these relations have been derived.

5.3.3 *Stress cycles in the steel*

In reinforced concrete and prestressed concrete the stress cycles in the steel need not be checked if, in the section considered, the stresses due to fluctuating loads are smaller than a certain percentage P_{max} *of the stresses due to the overall load. The percentage* P_{max} *is depen-*

Table A. Stress cycles in steel

grade of steel	*type of steel*	*description*	$\Delta\sigma_a$ *N/mm²*	P_{max} *%*
FeB 500	*HW*	*straight base material*	*200*	*65*
FeB 500	*HW*	*base material, hooks with radii* $\geq \phi_k$	*150*	*50*
FeB 500	*HW*	*flash butt welding*	*120*	*40*
FeB 500	*HW*	*cross welding*	*120*	*40*
FeB 500	*HK*	*straight base material*	*200*	*65*
FeB 500	*HK*	*base material, hooks with radii of curvature* $\geq \phi_k$	*150*	*50*
FeB 500	*HK*	*flash butt welding*	*120*	*40*
FeB 500	*HK*	*cross welding*	*120*	*35*

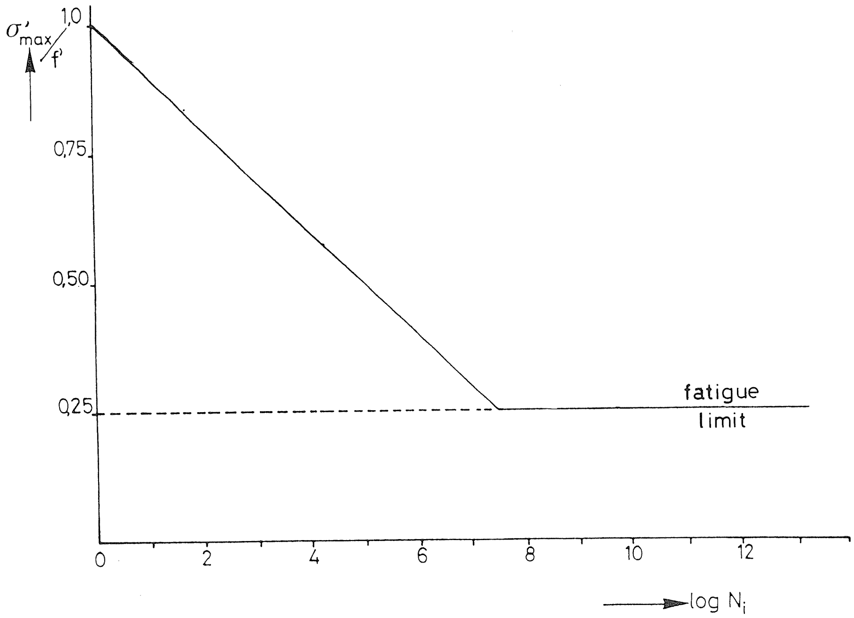


Fig. 17. Simplified design Wöhler curve.

dent on the type of steel and on the mode of fabrication. It can be read from Table A. In other cases a further check as to the fatigue limit state is needed.

Table A gives values for grade FeB 500 steel only. This is because steel manufacturers are expected shortly to discontinue the supply of lower grades.

The figures in the table have been taken from the "Draft Rules for the Design of Concrete Bridges" [23].

5.3.4 *Prestressed concrete*

When prestressed concrete is used the stress fluctuation in the steel need not be checked if the stress fluctuations in the section are absorbed by the concrete.

5.4 *Serviceability limit state*

In principle, the serviceability limit states, i.e. those concerning crack formation and deflection, need not be further examined.

The formulas for crack width and deflection used in design rules for concrete are based on experimental data obtained after some 1000 cycles of a service load. It has emerged from the various fatigue studies that within such a number of load cycles, the increase in the deformations is realized for the greater part. In the case of cyclic loading it is the first few hundred load cycles that contribute substantially to the greater deformations. The additional deformation occurring just before fatigue failure of the concrete is not very interesting in this context, because it takes place at a much higher load than that which is used in assessing the serviceability limit state. The possibility of such extra deformations occurring is covered by the assessment of the ultimate limit state.

If the design specifies that no crack formation should take place, the tensile stress in the concrete has to be checked for a combination of design values of the load based on the partial load factors γ_b *applying to the serviceability limit state.*

This condition may apply to structures that have to meet special requirements as to durability or to structures serving for the storage of gases or liquids.

5.5 *Load-bearing capacity*

It should be demonstrated that at the design values for loads and materials strengths the ultimate limit state is not exceeded.

To take into account the effect of non-constant-amplitude loads, use should be made of Miner's damage rule

$$\boxed{M_S < M_M}$$

where

M_S = *Miner sum, or the fatigue damage caused by the design value of the load*
M_M = *Miner number, or the criterion for fatigue failure*

For each of the base materials, concrete, reinforcing steel and prestressing steel, the Miner number should be

$$\boxed{M_M = 1.0}$$

The various MaTS studies have shown that the use of Miner's Rule for random loads leads to an adequate assessment of fatigue damage (i.e. of service life). The scatter in the results of such tests is due to the normal scatter in concrete strengths. In the present procedure this was taken into account in the definition of the design value of the concrete strength.

It has also emerged from these studies that the value of the Miner number is not always equal to 1.0. Deviations occur which are dependent on the parameters of the fluctuating load, on the one hand, and on the properties of the concrete used, on the other. Considering that the Miner number is a quantity with a log-normal character, these deviations may be said to be of relatively minor importance. Therefore the value of $M_M = 1.0$ has been assumed for the present procedure. Any undue optimism in this respect is taken to be compensated for by the other conservative assumptions within this procedure.

6 *Directions for design and execution*

Since fatigue is promoted by the presence of stress concentrations, it is recommended to design and to execute the structure in such a way as to avoid stress concentrations.
Damage to the reinforcing and the prestressing steel should be prevented. It is furthermore advisable to avoid fusion welding, or at least to carry it out with great care.
The anchorage zones of reinforcing and prestressing steel should be located outside areas which are prone to fatigue.
In the assessment of load-bearing capacity it is stated that the bond stress of reinforcing steel need not be examined. The preceding rule justifies this statement.
The simultaneous exposure of steel to fluctuating stresses and corrosion has a highly detrimental effect. Under such conditions adequate precautions should be taken against the penetration of carbondioxide or corrosive salts.

7 *Inspection and repairs*

It is recommended to inspect critical fatigue locations in a structure for damage to regular intervals (say, every 5 years).
Research so far has not yet produced an operational monitoring technique for concrete structures. An inspection will therefore in general have to be restricted to a visual examination of the concrete structure.
It has further been found that in the case of collapse through fatigue excessive deformations will occur only in the very last stage of service life. This implies that inspection will be meaningful in special cases only.
If a concrete structure needs any repairs, check calculations should be carried out to demonstrate that after repair the structure is sufficiently reliable. These calculations should indicate, among other things, to what extent the repair material will be involved in carrying the loads.

Part III Calculation examples

1 Introduction

In this part of the report two calculation examples will be given. The main goal is to demonstrate the general aspects of the procedure. Besides an insight in the relevance of the fatigue limit state of some realistic structures will be gained.
The two examples regard motorway viaducts:
- with a deck that is built in situ as a solid plate;
- with a deck that is built from prefab hollow beams.

From the static design calculation only the results will be mentioned. These are adopted from the original and realistic design of the two viaducts.

2 Viaduct in a motorway executed as a solid plate

2.1 *General*

In the Figs. 18 and 19 the longitudinal and the cross-section of the viaduct are given.

The resulting stresses of the static design are mentioned in Table 2. The original values of the mobile loading however are corrected. The static design was based on the load of two vehicles. For the fatigue analysis it is more realistic to start with one single vehicle. In general the viaduct will be loaded with only one heavy truck. In the table the distributed mobile loading that is caused by light weight vehicles such as cars, vans and motor bicycles has been made explicit. As can be seen from the table some loadings have a positive and a negative stress contribution, depending on the place of the load in

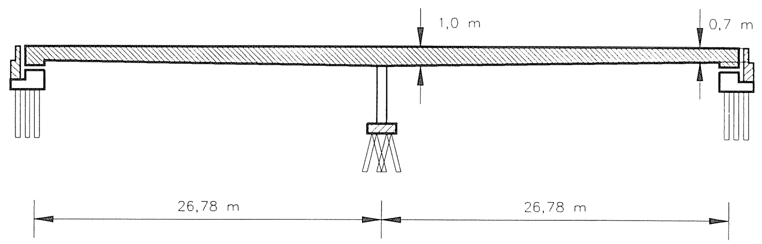


Fig. 18. Longitudinal section of the solid plate.

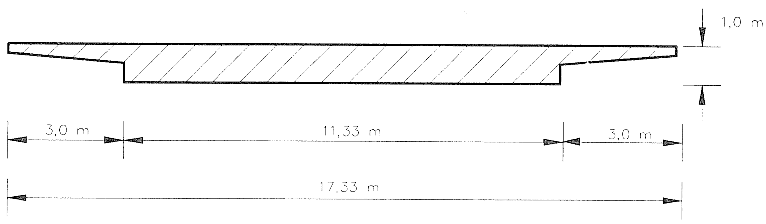


Fig. 19. Cross-section of the solid plate.

the longitudinal section. Positive means in this connection that the effects of the loading have the same sign as the effect of the dead weight.

2.2 *Loads and loading effects according to the fatigue procedure*

In the fatigue procedure some design values have other definitions than in the static design. Those values have to be recalculated:

Table 2. Stresses adopted from the static design procedure

number	notation	description	under fibre (kN/m^2)	upper fibre (kN/m^2)
1	G	dead weight	9331	−7767
2	R	permanent load	1551	−1291
3	Q_p	positive distributed traffic load	2798	−2329
		positive truck load	2934	−2442
4	Q_n	negative distributed traffic load	− 879	731
		negative truck load	− 583	485
5	T_p	positive temperature gradient	1640	−1365
6	T_n	negative temperature gradient	− 410	341
7	Z_p	positive settlement	182	− 152
8	Z_n	negative settlement	− 182	152
9	M_{av}	original prestressing moment	−14108	11742
10	N_{av}	original prestressing force	− 6627	−6627
11	-	loss of prestress 14.8 %	3069	− 757

Dead weight (1): nominal value (50 %) and $\gamma = 1.0$ (dead weight) gives:

$$\sigma_0 = 9331 \text{ kN/m}^2$$
$$\sigma_b = -7767 \text{ kN/m}^2$$

Static load (2): characteristic value (5 %) and $\gamma = 1.2$; in this case it is assumed that in the static design the characteristic value has been used; the design values for the fatigue analysis are:

$$\sigma_0 = 1.2 \cdot 1551 = 1861 \text{ kN/m}^2$$
$$\sigma_b = 1.2 \cdot -1291 = -1549 \text{ kN/m}^2$$

Traffic load (3, 4): with the load factor for fluctuating loads $\gamma = 1.3$ the following design values for the positive stresses due to distributed traffic can be calculated:

$$\sigma_0 = 1.3 \cdot 2798 = 3637 \text{ kN/m}^2$$
$$\sigma_b = 1.3 \cdot -2329 = -3028 \text{ kN/m}^2$$

The negative stresses are:

$$\sigma_0 = 1.3 \cdot -879 = -1143 \text{ kN/m}^2$$
$$\sigma_b = 1.3 \cdot 731 = 950 \text{ kN/m}^2$$

For the truck load the calculation is more complicated. The truck load must be related to the actual loading during the reference period. The static design is based on two truck loads of 600 kN. The simultaneous occurrence of two heavy trucks is relatively seldom. For the time being the calculation will be done for one truck load of 100 kN. This load must be considered as an unit load. On basis of statistic information of truck loads on motorways in The Netherlands the unit load will be converted to a stress spectrum. In this stage the load factor will be processed:

$$Q_{P,p} \rightarrow \sigma_0 = 1.3 \cdot 2934/6 = 636 \text{ kN/m}^2$$
$$\sigma_b = 1.3 \cdot -2442/6 = -529 \text{ kN/m}^2$$
$$Q_{P,n} \rightarrow \sigma_0 = 1.3 \cdot -583/6 = -126 \text{ kN/m}^2$$
$$\sigma_b = 1.3 \cdot 485/6 = 105 \text{ kN/m}^2$$

During the passage of a truck the stress in a certain fibre varies between the "positive" and the "negative" value. This characterises a stress cycle.

Temperature gradient (6): in the calculation it is assumed that during 1/4 of the reference period a temperature gradient is present. During the remaining 3/4 part of the effect of a temperature gradient has not to be taken into account. For the sake of simplicity only one value of the temperature gradient will be taken into account. This is the value from the static design. As the temperature gradient is a not permanent the fluctuating load factor is used $\gamma = 1.3$:

$$T_p \rightarrow \sigma_0 = 1.3 \cdot 1640 = 2132 \text{ kN/m}^2$$
$$\sigma_b = 1.3 \cdot -1365 = -1775 \text{ kN/m}^2$$
$$T_n \rightarrow \sigma_0 = 1.3 \cdot -410 = -533 \text{ kN/m}^2$$
$$\sigma_b = 1.3 \cdot 341 = 443 \text{ kN/m}^2$$

This stress fluctuates in reality during 24 hours between the value zero and the specified value. This effect has to be combined with the traffic load.

Settlement (7, 8): this effect will not be taken into account because the influence is in this case relatively small.

Prestressing (9): the period in which the loss of prestress is achieved is short in relation to the reference period. Because of this aspect it is assumed that the loss of prestress is present after the construction period. During the service life the design stresses can be based on the reduced prestressing force:

prestressing moment $\sigma_0 = (1 - 0.148) \cdot -14108 = -12020 \text{ kN/m}^2$

$\sigma_b = (1 - 0.148) \cdot 11742 = 10004 \text{ kN/m}^2$

prestressing force $\sigma_0 = \sigma_b = (1 - 0.148) \cdot 6627 = -5646 \text{ kN/m}^2$

In Table 3 the design stresses for the fatigue analysis are given.

Table 3. Design stresses for the fatigue analysis

number	notation	description	under fibre (kN/m^2)	upper fibre (kN/m^2)
1	G	dead weight	9331	–7767
2	R	static load	1861	–1549
3	Q_p	positive distributed traffic load	3637	–3028
		positive unit truck load	636	– 529
4	Q_n	negative distributed traffic load	– 1134	950
		negative unit truck load	– 126	105
5	T_p	positive temperature gradient	2132	–1775
6	T_n	negative temperature gradient	– 533	443
9	M_v	prestressing moment	–12020	10004
10	N_v	prestress force	– 5646	–5646

In this stage the effects of the dead load and the permanent load can be combined to a design value B_p:

$$B_p = G + R + Q_p + M_v + N_v$$

This results in combined stresses:

$$\sigma_0 = 9331 + 1861 + 3637 - 12020 - 5646 = -2837 \qquad \sigma_b = -7767 - 1549 - 3028 + 10004 - 5646 = -7986$$

In this way the stresses from Table 3 can be simplified to the values in Table 4.

Table 4. Design values of the stresses in the fatigue analysis

number	notation	description	under fibre (kN/m^2)	upper fibre (kN/m^2)
a	B_p	permanent load	–2837	–7986
b	T_p	positive temperature gradient	2132	–1775
c	T_n	negative temperature gradient	– 533	443
d	L_p	positive unit truck load 100 kN	636	– 529
e	L_n	negative unit truck load 100 kN	– 126	105

2.3 *Traffic load*

On several places in the Dutch motorways registrations of the traffic load have been made. These registrations are not ment for the fatigue analysis of concrete viaducts. They are directed on the recording of axle loads. In the present calculations example we need the spectrum of truck loads (weight of the truck and the cargo). This implies the revision of the axle load registration. The result of that for a period of 50 years is given in Table 5.

Vehicle loads smaller than 100 kN are not registrated in the table. The loading effect of these trucks is assumed to be comprised in the effect of the distributed traffic load. The fatigue analysis wil be based on the mean value of truck load from a loading class and the number of vehicles in that class. It is assumed that the highest truck load is equal to 1200 kN.

Table 5. Loads of segmented and non-segmented trucks

truck load	segmented	number non-segmented	total
< 100 kN	$1.5 \cdot 10^6$	$26.1 \cdot 10^6$	$27.6 \cdot 10^6$
100– 200	$13.1 \cdot 10^6$	$10.0 \cdot 10^6$	$23.3 \cdot 10^6$
200– 300	$11.6 \cdot 10^6$	$1.6 \cdot 10^6$	$13.2 \cdot 10^6$
300– 400	$6.9 \cdot 10^6$	$13.2 \cdot 10^4$	$6.9 \cdot 10^6$
400– 500	$5.3 \cdot 10^6$	$1.9 \cdot 10^4$	$5.3 \cdot 10^6$
500– 600	$2.3 \cdot 10^6$	–	$2.3 \cdot 10^6$
600– 700	$92.5 \cdot 10^4$	–	$92.5 \cdot 10^4$
700– 800	$25.2 \cdot 10^4$	–	$25.2 \cdot 10^4$
800– 900	$2.9 \cdot 10^4$	–	$2.9 \cdot 10^4$
900–1000	$2.1 \cdot 10^4$	–	$2.1 \cdot 10^4$
1000–1100	$2.1 \cdot 10^4$	–	$2.1 \cdot 10^4$
1100–1200	$0.8 \cdot 10^4$	–	$8.2 \cdot 10^4$
> 1200 kN	$0.4 \cdot 10^4$	–	$0.4 \cdot 10^4$

2.4 *Temperature load*

As mentioned before, the temperature effect is present during 25 % of the time. For that period the combined influence of the traffic load and the temperature gradient has to be considered.

The temperature gradient is not constant during the day. To account for that effect the day will be devided in six periods of 4 hours. In doing this it is also necessary to account for the aspect that the traffic load is not equally distributed over the day. From the axle load registrations the distribution of the truck loads over the day from Table 6 can be derived. In Tabel 7 the truck load spectrum for the combination of the number of truck loads with $n_{\text{with T}}$ and the number without a temperature gradient $n_{\text{without T}}$ is given.

Table 6. Truck load distribution over one day

period	number of trucks	percentage
0– 4 o'clock	163	2
4– 8 o'clock	1769	19
8–12 o'clock	2598	27
12–16 o'clock	2646	28
16–20 o'clock	1779	19
20–24 o'clock	480	5

Table 7. Truck load distribution for the fatigue analysis

mean load	$n_{\text{without T}}$	$n_{\text{with T}}$	considered period of the day 0–4	4–8	8–12	12–16	16–20	20–24
75	$20.7 \cdot 10^6$	$6.9 \cdot 10^6$	0.14	1.31	1.86	1.93	1.32	$0.35 \cdot 10^6$
150	$17.3 \cdot 10^6$	$5.7 \cdot 10^6$	0.11	1.08	1.54	1.60	1.08	$0.29 \cdot 10^6$
250	$9.9 \cdot 10^6$	$3.3 \cdot 10^6$	0.07	0.63	0.89	0.92	0.63	$0.17 \cdot 10^6$
350	$5.2 \cdot 10^6$	$1.7 \cdot 10^6$	0.03	0.32	0.46	0.48	0.32	$0.09 \cdot 10^6$
450	$4.0 \cdot 10^6$	$1.3 \cdot 10^6$	0.03	0.25	0.35	0.36	0.25	$0.07 \cdot 10^6$
550	$1.7 \cdot 10^6$	$0.6 \cdot 10^6$	0.01	0.11	0.16	0.16	0.11	$0.03 \cdot 10^6$
650	$69.4 \cdot 10^4$	$23.1 \cdot 10^4$	0.46	4.39	6.24	6.47	4.39	$1.15 \cdot 10^4$
750	$18.9 \cdot 10^4$	$6.3 \cdot 10^4$	0.13	1.20	1.70	1.76	1.20	$0.31 \cdot 10^4$
850	$2.2 \cdot 10^4$	$0.7 \cdot 10^4$	0.01	0.13	0.19	0.20	0.13	$0.04 \cdot 10^4$
950	$1.6 \cdot 10^4$	$0.5 \cdot 10^4$	0.01	0.10	0.14	0.14	0.10	$0.03 \cdot 10^4$
1050	$1.6 \cdot 10^4$	$0.5 \cdot 10^4$	0.01	0.10	0.14	0.14	0.10	$0.03 \cdot 10^4$
1150	$0.6 \cdot 10^4$	$0.2 \cdot 10^4$	0.004	0.038	0.054	0.056	0.038	$0.010 \cdot 10^4$
1200	$0.3 \cdot 10^4$	$0.1 \cdot 10^4$	0.002	0.019	0.027	0.028	0.019	$0.005 \cdot 10^4$

In this example a maximum temperature gradient of 15 °C is adopted. The maximum value is reached in the period from 12 to 16 o'clock. The maximum value is gradually be brought about during the day. In Table 8 the temperature distribution and the resulting stresses are given. For the sake of simplicity only the positive gradient from Table 4 is taken into account.

Table 8. Temperature distribution during one day

period	temperature difference	under stress σ_0 (kN/m^2)	upper stress σ_b (kN/m^2)
0– 4 o'clock	0 °C	0	0
4– 8 o'clock	5 °C	711	− 592
8–12 o'clock	10 °C	1421	−1183
12–16 o'clock	15 °C	2132	−1775
16–20 o'clock	10 °C	1421	−1183
20–24 o'clock	0 °C	711	− 592

As the temperature distribution is symmetric over the day, it is possible to take together the information from Table 7 and Table 8 to the information of Table 9.

2.5 *Number and magnitude of the cycles*

For the sake of simplicity only the upper stress will be considered. For the final calculations the other limit states have also to be taken into account.
In Table 10 a survey of the stresses due to the truck loads is given. The temperature effect is not yet taken into consideration.

Table 9. Truck load distribution with temperature influence

mean load (kN)	$n_{0\,°C}$	$n_{5\,°C}$	$n_{10\,°C}$	$n_{15\,°C}$
75	$20.7 \cdot 10^6$	$1.66 \cdot 10^6$	$3.18 \cdot 10^6$	$1.93 \cdot 10^6$
150	$17.3 \cdot 10^6$	$1.37 \cdot 10^6$	$2.62 \cdot 10^6$	$1.60 \cdot 10^6$
250	$9.9 \cdot 10^6$	$0.80 \cdot 10^6$	$1.52 \cdot 10^6$	$0.92 \cdot 10^6$
350	$5.2 \cdot 10^6$	$0.41 \cdot 10^6$	$0.78 \cdot 10^6$	$0.46 \cdot 10^6$
450	$4.0 \cdot 10^6$	$0.32 \cdot 10^6$	$0.78 \cdot 10^6$	$0.48 \cdot 10^6$
550	$1.7 \cdot 10^6$	$0.14 \cdot 10^6$	$0.27 \cdot 10^6$	$0.16 \cdot 10^6$
650	$69.9 \cdot 10^4$	$5.54 \cdot 10^4$	$10.63 \cdot 10^4$	$6.47 \cdot 10^4$
750	$19.0 \cdot 10^4$	$1.51 \cdot 10^4$	$2.90 \cdot 10^4$	$1.76 \cdot 10^4$
850	$2.2 \cdot 10^4$	$0.17 \cdot 10^4$	$0.32 \cdot 10^4$	$0.20 \cdot 10^4$
950	$1.6 \cdot 10^4$	$0.13 \cdot 10^4$	$0.24 \cdot 10^4$	$0.14 \cdot 10^4$
1050	$1.6 \cdot 10^4$	$0.13 \cdot 10^4$	$0.24 \cdot 10^4$	$0.14 \cdot 10^4$
1150	$0.6 \cdot 10^4$	$0.048 \cdot 10^4$	$0.092 \cdot 10^3$	$0.056 \cdot 10^4$
1200	$0.3 \cdot 10^4$	$0.024 \cdot 10^4$	$0.46 \cdot 10^4$	$0.028 \cdot 10^4$

Table 10. Upper stress without the temperature effect

load	$n_{\text{without T}}$	σ_b
75	$20.7 \cdot 10^6$	$-7986-529 \cdot 0.75 = -8381$
150	$17.3 \cdot 10^6$	$-7986-529 \cdot 1.5 = -8780$
250	$9.9 \cdot 10^6$	$-7986-529 \cdot 2.5 = -9309$
350	$5.2 \cdot 10^6$	$-7986-529 \cdot 3.5 = -9838$
450	$4.0 \cdot 10^6$	$-7986-529 \cdot 4.5 = -10367$
550	$1.7 \cdot 10^6$	$-7986-529 \cdot 5.5 = -10896$
650	$69.4 \cdot 10^4$	$-7986-529 \cdot 6.5 = -11425$
750	$18.9 \cdot 10^4$	$-7986-529 \cdot 7.5 = -11953$
850	$2.2 \cdot 10^4$	$-7986-529 \cdot 8.5 = -12483$
950	$1.6 \cdot 10^4$	$-7986-529 \cdot 9.5 = -13012$
1050	$1.6 \cdot 10^4$	$-7986-529 \cdot 10.5 = -13540$
1150	$0.6 \cdot 10^4$	$-7986-529 \cdot 11.5 = -14070$
1200	$0.3 \cdot 10^4$	$-7986-529 \cdot 12.0 = -14334$

In Table 11 the information of Table 10 is combined with the temperature effect.

Table 11. Stress spectrum

		σ'_{max} at a temperature difference of			
load	σ'_{min}	0 °C	5 °C	10 °C	15 °C
75	−7986	− 8381	− 8973	− 9565	−10157
150	−7986	− 8750	− 9372	− 9964	−10556
250	−7986	− 9309	− 9901	−10493	−11085
350	−7986	− 9838	−10430	−11022	−11614
450	−7986	−10367	−10959	−11551	−12143
550	−7986	−10896	−11488	−12080	−12672
650	−7986	−11425	−12017	−12609	−13201
750	−7986	−11953	−12545	−13137	−13729
850	−7986	−12483	−13075	−13667	−14259
950	−7986	−13012	−13604	−14196	−14788
1050	−7986	−13540	−14132	−14724	−15316
1150	−7986	−14070	−14662	−15254	−15846
1200	−7986	−14334	−14926	−15518	−16110

The characteristic compressive strength for concrete grade B 37.5 is according to the Dutch concrete code equal to 30.0 N/mm². The fatigue compressive strength f'_{bkv} is in this case also equal to:

$$f'_{bkv} = 30.0 \text{ N/mm}^2 = 30{,}000 \text{ kN/m}^2$$

The design value for the fatigue compressive strength for flexure with a prestressing force is equal to:

$$f'_{dv} = f'_{bkv}/\gamma_m = 30.0/1.25 = 24.0 \text{ N/mm}^2 = 24{,}000 \text{ kN/m}^2$$

On basis of this value Table 11 can be converted to stresses relative to the fatigue strength. The results are given in Table 12.

Table 12. Stress spectrum expressed as relative stresses

mean load	σ'_{min}/f'_{dv}	σ'_{max}/f'_{dv} at a temperature difference 0 °C	5 °C	10 °C	15 °C
75	−0.33	−0.35	−0.37	−0.40	−0.42
150	−0.33	−0.37	−0.39	−0.42	−0.44
250	−0.33	−0.39	−0.41	−0.44	−0.46
350	−0.33	−0.41	−0.43	−0.46	−0.48
450	−0.33	−0.43	−0.46	−0.48	−0.51
550	−0.33	−0.45	−0.48	−0.50	−0.53
650	−0.33	−0.48	−0.50	−0.53	−0.55
750	−0.33	−0.50	−0.52	−0.55	−0.57
850	−0.33	−0.52	−0.54	−0.57	−0.59
950	−0.33	−0.54	−0.57	−0.59	−0.62
1050	−0.33	−0.56	−0.59	−0.61	−0.64
1150	−0.33	−0.59	−0.61	−0.64	−0.66
1200	−0.33	−0.60	−0.62	−0.65	−0.67

This table is converted in Table 13 to maximum stresses and stress ratios.

Table 13. Stress spectrum expressed in σ'_{max}/f'_{dv} and R

mean load	0 °C		5 °C		10 °C		15 °C	
	σ/f	R	σ/f	R	σ/f	R	σ/f	R
75	−0.35	0.94	−0.37	0.89	−0.40	0.83	−0.42	0.79
150	−0.37	0.89	−0.39	0.85	−0.42	0.79	−0.44	0.75
250	−0.39	0.85	−0.41	0.80	−0.44	0.75	−0.46	0.72
350	−0.41	0.80	−0.43	0.77	−0.46	0.72	−0.48	0.69
450	−0.43	0.77	−0.46	0.72	−0.48	0.69	−0.51	0.65
550	−0.45	0.73	−0.48	0.69	−0.50	0.66	−0.53	0.62
650	−0.48	0.69	−0.50	0.66	−0.53	0.62	−0.55	0.60
750	−0.50	0.66	−0.52	0.63	−0.55	0.60	−0.57	0.58
850	−0.52	0.63	−0.54	0.61	−0.57	0.58	−0.59	0.56
950	−0.54	0.61	−0.57	0.58	−0.59	0.56	−0.62	0.53
1050	−0.56	0.59	−0.59	0.56	−0.61	0.54	−0.64	0.52
1150	−0.59	0.56	−0.61	0.54	−0.64	0.52	−0.66	0.50
1200	−0.60	0.55	−0.62	0.53	−0.65	0.51	−0.67	0.49

On basis of the design Wöhler curve from the fatigue procedure it is now possible to calculate the value log N_i. The results are given in Table 14.

Table 14. Values of log N_i following from the spectrum in Table 12

mean load	temperature difference 0 °C	5 °C	10 °C	15 °C
75	26.5	19.0	14.6	12.7
150	19.0	15.8	12.7	11.2
250	15.8	13.2	11.2	10.2
350	13.2	11.9	10.2	9.3
450	11.9	10.2	9.3	8.3
550	10.6	9.3	8.6	7.6
650	9.3	8.6	7.6	7.1
750	8.6	7.9	7.1	6.6
850	7.9	7.4	6.6	6.2
950	7.4	6.6	6.2	5.5
1050	6.9	6.2	5.8	5.2
1150	6.2	5.8	5.2	4.8
1200	6.0	5.5	5.0	4.6

The contributions to the Miner sum are given in Table 15.

Table 15. Contributions to the Miner sum

mean load	temperature difference 0 °C	5 °C	10 °C	15 °C
75	0.0000	0.0000	0.0000	0.0000
150	0.0000	0.0000	0.0000	0.0000
250	0.0000	0.0000	0.0001	0.0001
350	0.0000	0.0000	0.0000	0.0002
450	0.0000	0.0000	0.0004	0.0024
550	0.0000	0.0001	0.0007	0.0040
650	0.0004	0.0001	0.0027	0.0051
750	0.0005	0.0002	0.0023	0.0044
850	0.0003	0.0001	0.0008	0.0013
950	0.0006	0.0008	0.0015	0.0044
1050	0.0020	0.0008	0.0038	0.0088
1150	0.0038	0.0008	0.0058	0.0089
1200	0.0030	0.0008	0.0460	0.0070
Σ	0.0108	0.0037	0.0641	0.0466
Miner sum: 0.1252				

This calculation shows that the Miner sum is smaller than the ultimate value 1. This means that the ultimate fatigue limit state is not reached.

3 Viaduct in a motorway built from hollow prefab beams

3.1 *General*

The longitudinal and the cross-section of the viaduct are given in the Figs. 20 and 21.

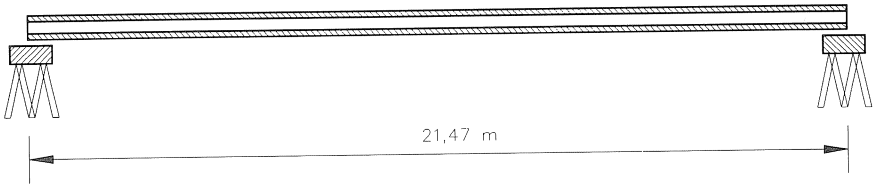


Fig. 20. Longitudinal section.

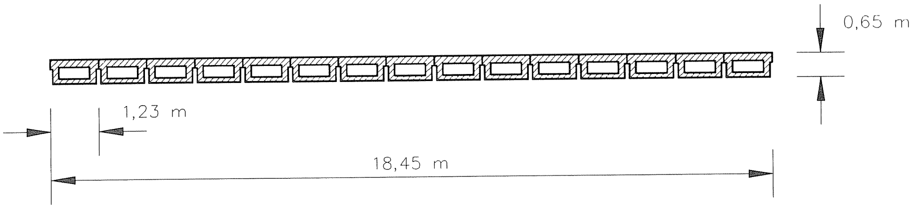


Fig. 21. Cross-section.

The fatigue procedure is related to one of the edge beams. The results of the original static design for that beam are given in Table 16.

Table 16. Results of the original static design

number	notation	description	under stress (N/mm^2)	upper stress (N/mm^2)
1	G_b	dead weight beam	10.08	− 9.48
2	G_d	dead weight compression layer	2.07	− 1.81
3	R	static load	3.59	− 2.68
4	Q	traffic load	15.72	−11.71
5	FA	begin prestress	−34.80	8.20
6	F	resulting prestress	−30.08	7.09

3.2 *Loads and loading effects according to the fatigue procedure*

The design loads from this original static procedure have to be converted according to the fatigue procedure.

Dead weight of the beam (1): nominal value (50 %) and a load factor for dead weight $\gamma = 1.0$:

$$\sigma_0 = 10.08 \text{ N/mm}^2$$
$$\sigma_b = 9.48 \text{ N/mm}^2$$

Dead weight compression layer (2): nominal value (50 %) and $\gamma = 1.0$ (dead weight):

$$\sigma_0 = 2.07 \text{ N/mm}^2$$
$$\sigma_b = -1.81 \text{ N/mm}^2$$

Static load (3): characteristic value (5 %) and $\gamma = 1.2$. In this case it is assumed that in the static load the characteristic load has been used:

$$\sigma_0 = 1.2 \cdot \quad 3.59 = \quad 4.31 \text{ N/mm}^2$$
$$\sigma_b = 1.2 \cdot -2.68 = -3.22 \text{ N/mm}^2$$

Traffic load (4): the distributed traffic load gives a maximum moment of 317.13 kNm. This has to be multiplied with a load factor 1.3. At the same time a correction for the stiffness has to be made. The original calculation was based on the assumption that all the beams were equally stiff. The edge beam has however a lower stiffness:

$$\sigma_0 = \quad 4.77 \cdot 16720/18774 \cdot 1.3 = \quad 5.52 \text{ N/mm}^2$$
$$\sigma_b' = -3.56 \cdot 16720/18774 \cdot 1.3 = -4.12 \text{ N/mm}^2$$

The stresses due to a truck with a total load of 600 kN will be interpreted as the unit load. The load factor is 1.3:

$$\sigma_0 = 1.3 \cdot \quad 10.97 = \quad 14.26 \text{ N/mm}^2$$
$$\sigma_b = 1.3 \cdot - \quad 8.17 = -10.63 \text{ N/mm}^2$$

Prestressing (5, 6): the prestressing is taken into account including the losses of the prestress, like it has been done with the previous example. The load factor for the prestressing is $\gamma = 1.0$:

$$\sigma_0 = 1.0 \cdot -30.08 = -30.08 \text{ N/mm}^2$$
$$\sigma_b = 1.0 \cdot \quad 7.09 = \quad 7.09 \text{ N/mm}^2$$

The resulting basis stresses for the fatigue analysis are given in Table 17.

Table 17. Design values for the fatigue analysis

number	notation	description	under stress (N/mm^2)	upper stress (N/mm^2)
1	G	dead weight	12.15	−11.29
2	R	static load	4.31	− 3.22
3	Q_p	mobile distributed load	5.52	− 4.12
		unit truck load 600 kN	14.26	−10.63
4	V	prestress	−30.08	7.09

In this stage the dead weight and the static load can be combined to a design value of the permanent load B_p, according to:

$$B_p = G + R + Q_p + V$$

This gives resulting stresses with a magnitude:

$$\sigma_0 = \quad 12.15 + 4.31 + 5.52 - 30.08 = - \quad 8.10 \text{ N/mm}^2$$
$$\sigma_b = -11.29 - 3.22 - 4.12 + \quad 7.09 = -11.54 \text{ N/mm}^2$$

3.3 *Traffic load*

For the calculation the traffic loads from table 5 have been used. In combination with the results from Table 18 it is possible to calculate the maximum under and upper stresses. In this case it is however necessary to make correction for the length of the passing trucks. This is necessary because of the fact that length of the viaduct is small in relation to the length of the trucks. In this case the following correction factors have been used:

truck load < 100 kN	correction factor 0.99
truck load 100–200 kN	correction factor 0.89
truck load 200–300 kN	correction factor 0.82
truck load > 300 kN	correction factor 0.80

Table 18. Load spectrum based on design values

load (kN)	n	σ'_b (N/mm^2) minimum	maximum	ratio
75	$27.6 \cdot 10^6$	−11.54	−12.86	0.90
150	$23.3 \cdot 10^6$	−11.54	−13.91	0.83
250	$13.2 \cdot 10^6$	−11.54	−15.77	0.73
350	$6.9 \cdot 10^6$	−11.54	−16.50	0.70
450	$5.3 \cdot 10^6$	−11.54	−17.92	0.64
550	$2.3 \cdot 10^6$	−11.54	−19.34	0.60
650	$92.5 \cdot 10^4$	−11.54	−20.75	0.56
750	$25.2 \cdot 10^4$	−11.54	−22.17	0.52
850	$2.9 \cdot 10^4$	−11.54	−23.59	0.49
950	$2.1 \cdot 10^4$	−11.54	−25.00	0.46
1050	$2.1 \cdot 10^4$	−11.54	−26.42	0.44
1150	$0.8 \cdot 10^4$	−11.54	−27.84	0.41
1200	$0.4 \cdot 10^4$	−11.54	−28.55	0.40

A concrete grade B 52.5 has according to the Dutch Concrete Code a characteristic strength $f'_{bk} = 42.0$ N/mm^2. The fatigue compressive strength f'_{bkv} is in that case:

$$f'_{bkv} = \frac{(42.0 - 30.0)}{2} + 30.0 = 36.0 \text{ N/mm}^2$$

For flexure with prestressing the design compressive strength follows from:

$$f'_{dv} = f'_{bkv}/\gamma_m = 36.0/1.25 = 28.8 \text{ N/mm}^2$$

On basis of this value Table 18 can be converted to the relative stresses from Table 19. With the design Wöhler curve from the fatigue procedure it is possible to calculate the log N_i values and the contributions to the Miner sum M_S that also are mentioned in this table.

Table 19. Stress spectrum expressed in relative stresses

mean load	n	σ'_{max}/f'_{dv}	R	$\log N_i$	M_S
75	$27.6 \cdot 10^6$	0.45	0.90	17.9	0.000
150	$23.3 \cdot 10^6$	0.48	0.83	12.6	0.000
250	$13.2 \cdot 10^6$	0.53	0.73	9.59	0.003
350	$6.9 \cdot 10^6$	0.57	0.70	7.85	0.097
450	$5.3 \cdot 10^6$	0.62	0.64	6.33	2.479
550	$2.3 \cdot 10^6$	0.67	0.60	5.22	13.86
650	$92.5 \cdot 10^4$	0.72	0.56	4.22	55.74
750	$25.2 \cdot 10^4$	0.77	0.52	3.32	148.4
850	$2.9 \cdot 10^4$	0.82	0.49	2.52	87.58
950	$2.1 \cdot 10^4$	0.87	0.46	1.77	356.6
1050	$2.1 \cdot 10^4$	0.92	0.44	1.07	1787.4
1150	$0.8 \cdot 10^4$	0.97	0.41	0.39	3259.0
1200	$0.4 \cdot 10^4$	0.99	0.40	0.13	2965.2
					$\Sigma M_S = 8676.4$

The Miner sum is in this case amply larger than the ultimate value 1. From the fatigue analysis it must be concluded that the reliability of this structure is too low.
In the fatigue analysis use has been made from partial safety factors with magnitudes that have been calibrated from static design procedures. In this respect it was interesting to recalculate the viaduct with other sets of partial safety factors. The results are given in table 20. It is clear from that results that the Miner sum is primarily dependent of the magnitude of the relative stresses.

Table 20. Miner sums belonging to different sets of safety factors

material γ_m	permanent γ_p	static γ_r	mobile γ_w	Miner sum M_S
1.25	1.0	1.2	1.3	8676.4
1.25	1.0	1.0	1.0	28.94
1.0	1.0	1.0	1.0	0.266

In the two calculation examples of the viaducts the total mobile loading consists of a mobile distributed load and the truck loads. This is in analogy to the dutch code for bridge design. From daily experience it can be concluded that this combination is not common. The original calculation (see the tables 18 and 19) is therefor repeated without the effect of the distributed mobile load. This implies that the maximum and minimum stresses are lowered with a value 4.12 N/mm^2. In that case the Miner sum is equal to 62.2, which is still larger than the ultimate value 1.

The fatigue analysis was based on traffic data from 1985. During the reference period of the viaduct an increase can be expected from:
- the number of vehicles; this influence is relatively small;
- the weight and the cargo of vehicles.

4 Conclusion

From the calculation example it is learned that it is in general possible to use the proposed procedure to determine the fatigue effect in concrete structures. Some data need however improvement, such as the traffic load distribution and the temperature effect.
Regarding the traffic load it is necessary to dispose of both axle load and vehicle load distributions. In that way it is possible to take into account the effect of small span and large span viaducts. Furthermore a better insight in the possibility of the presence of two or more heavy trucks at the same time on relevant places on a viaduct is necessary. In the presented examples this possibility has not been considered.
Concerning the temperature effect it is necessary to have information on the magnitude and the number of days on which the temperature effect is present.
The calculation of the hollow beam viaduct showed that the ultimate fatigue limit state is reached. This implies that it is necessary to intensify the number of example calculations for more limit states and not only for motorway viaducts but also for other type of structures that are subjected to fluctuating loads. In this respect it is also necessary to give a better basis to the load and material factors that have been proposed in the fatigue procedure. One of the best possibilities in this respect is to make probabilistic calculations of a variety of structures. An other possibility is to evaluate damaged structures on the presence of fatigue damage.

The author of this report welcomes any results from fatigue calculations that can improve the presented fatigue procedure.

References

1. LEEUWEN, J. VAN and A. J. M. SIEMES, Onderzoek vermoeiing van beton – eindrapport en veiligheidsbeschouwing (Investigations into the fatigue of concrete, Final report and safety aspects), IRO-StuPOC reports 3.3, 1977 (in Dutch).
2. SIEMES, A. J. M., Vermoeiing van beton, deel 1: Drukspanningen (Fatigue of concrete, Part 1: Compressive stresses), CUR-VB report 112 in co-operation with MaTS-IRO, Published by Betonvereniging Zoetermeer, 1983, p. 80 (in Dutch).
3. CORNELISSEN, H. A. W., Vermoeiing van beton, deel 2: Trek- en trek/drukspanningen (Fatigue of concrete, Part 2: Tensile and tensile/compressive stresses, CUR report 116 in co-operation with MaTS-IRO, Published by Betonvereniging Zoetermeer, 1984 (in Dutch).
4. CORNELISSEN, H. A. W., Vermoeiing van beton, deel 3: (Fatigue in concrete, Part 3:), CUR report 137 in co-operation with MaTS-IRO, Published by CUR Gouda, 1988 (in Dutch).
5. SIEMES, A. J. M., Vermoeiing van beton, deel 4: Random drukspanningen (Fatigue of concrete, Part 4: Random compressive stresses), CUR report 000 in co-operation with MaTS-IRO, Published by CUR Gouda, 1989 (in Dutch).
6. Design procedure for cement concrete roads (in Dutch), Published by VNC, 's-Hertogenbosch, 1985.
7. Rules for Concrete 1974/1982, Dutch Standard NEN 3880, Published by NNI, Delft, June 1982.
8. Rules for the Design, Construction and Inspection of Offshore Structures (with corrections), Published by DNV, Oslo, 1980.

9. Regulations for structural design of loadbearing structures intended for exploitation of petroleum resources (unofficial translation), Published by Norwegian Petroleum Directorate NPD, 1985.
10. FIP Recommendations: Design and construction of concrete sea structures, Published by Thomas Telfor Ltd., London, 1985.
11. WAAGAARD, K., Fatigue Strength Evaluation of Offshore Concrete Structures, IABSE Periodica 4/1982; IABSE Proceedings P-56/82, p. 97–115, 1982.
12. ZALESKI-ZAMENHOF, L. C., Analyse des effects des sollicitations répétitives sur les structures marines en béton (Analysis of the effects of loading cycles on concrete offshore structures), Colloque International sur la tenue des ouvrages en béton en mer. Acte de colloque no. 11, p. 16, 1980 (in French).
13. LEEUWEN, J. VAN and A. J. M. SIEMES, Miner's Rule with Respect to Plain Concrete, Heron, Vol. 24, Delft, 1979.
14. ZALESKI-ZAMENHOF, L. C., Fatigue Analysis Methodology with Respect to Prestressed Concrete Offshore Structures, Proceedings of the 4th International Symposium on Offshore Engineering at Coppe, Federal University of Rio de Janeiro, Brazil, 1983.
15. FURTAK, K., Ein Verfahren zur Berechnung der Betonfestigkeit unter Schwellenden Belastung (Method for the calculation of the concrete strength under cyclic loading), Cement and Concrete Research, Vol. 14, 1984.
16. MONNIER, TH., Fatigue Strength Procedure for Concrete Offshore Structures, TNO-IBBC report B-75-304/04.2.3043, 1975.
17. WARNER, R. F., Design of Concrete Structures for Fatigue Reliability, Bulletin of the Disaster Prevention Research Institute, Vol. 35, Part 2, June 1985.
18. SIEMES, A. J. M., Veiligheid van bouwconstructies – een probabilistische benadering (Safety of structures – A probabilistic approach), CUR-VB report 109, Published by Betonvereniging Zoetermeer, 1982 (in Dutch).
19. KARDENIS, H., S. VAN MAANEN and A. VROUWENVELDER, Probabilistic Reliability Analysis for the Fatigue Limit State of Gravity and Jacket-type Structures, 3th International Boss Conference in Boston, 1979.
20. GOSTELIE, E. M., Probabilistische vermoeiingsanalyse van een betonnen offshore constructie (Probabilistic fatigue analysis of a concrete offshore structure), Report on final subject, Applied Mechanics Section, Study Group for Offshore Engineering, Delft University of Technology, 1985.
21. SIEMES, A. J. M., Oriënterend onderzoek naar de mechanische eigenschappen van drie nieuwe betonsoorten (Exploratory investigation into the mechanical properties of three new types of concrete), TNO-IBBC report B-89-1989 (in Dutch).
22. LEEWIS, M. and H. E. VAN DER MOST, De dikte van ongewapende betonverhardingen (Thickness of plain concrete road carpets), Betonwegen Nieuws, number 43, December 1980.
23. CUR-VB Regulations Committee 6 "Concrete bridges", Concept Voorschriften Betonnen Bruggen (Draft Rules for Concrete Bridges), Published by NNI, Delft, December 1988 (in Dutch).
24. CORNELISSEN, H. A. W. and M. LEEWIS, Vermoeiing van ongewapende betonverhardingen; dimensionering en onderzoek (Fatigue of plain-concrete road carpets; dimensioning and research), Betonwegen Nieuws, no. 55, April 1984 (in Dutch).
25. Fatigue of Concrete Structures, draft 5, International working paper CEB, October 1987.
26. LEEUWEN, J. VAN, Sterkte en stijfheid van kolommen bij wisselbelasting; kolommen met een rechthoekige doorsnede (Strength and stiffness of columns under cyclic loads, Columns with a rectangular cross-section), CUR report 82A, Published by Betonvereniging Zoetermeer, 1976 (in Dutch).
27. LEEUWEN, J. VAN, Sterkte en stijfheid van kolommen bij wisselbelasting; kolommen met een cirkelvormige massieve doorsnede (Strength and stiffness of columns under cyclic loads; Columns with a circular, solid cross-section), CUR report 82B, Published by Betonvereniging Zoetermeer, 1977.