HERON contains contributions based mainly on research work performed in I.B.B.C. and STEVIN and related to strength of materials and structures and materials science.

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HERON vol. 23 1978 no. 2

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COMPOSITE BEAMS

Publications in HERON since 1970

Preface

The setting up, in 1970, of CUR Committee C 19 "Co-operation of precast beams with in-situ concrete" was inspired by the wish to stimulate the practical application of "concrete-concrete" composite beams. At the time, not much was known concerning the behaviour of such beams, while a suitable procedure for their analysis was virtually non-existent.

The Committee began its work by undertaking a literature research, which was completed in 1971. On the basis of that research a proposal for a simple set of rules was submitted to the then existing committee for codes and standards. The literature research also served as a starting point for the experimental investigations comprising the testing of 37 composite reinforced concrete beams, which was carried out in 1972 and 1973.

Although a draft final report was prepared as long ago as 1973, it was some considerable time before the results of the tests could be published in this report. The main cause of this delay was the fact that a new code of practice for concrete construction (VB 1974) had meanwhile been issued and that it was desirable, from the outset, to bring the practical recommendations for the design and analysis of composite beams into line with that code.

The Committee was constituted as follows:

prof. ir. J. W. Kamerling, chairman
ir. A. van den Beukel, secretary
ir. H. W. Beumer
ir. M. Dragosavić
ir. A. Krijgsman
ir. H. J. C. Oud
dr. ir. G. Scherpbier, mentor

The research was carried out at the Institute TNO for Building Materials and Building Structures by ir. A. van den Beukel, who is also the author of this report.

Thanks are due to the Netherlands Committee for Concrete Research (CUR) for financing this work.

The translation from the original CUR Report No. 86 "Samengestelde liggers" (in Dutch) into English has been prepared by ir. C. van Amerongen, MICE.

COMPOSITE BEAMS

Summary

Composite beams present a number of aspects which had to be further investigated with a view to arriving at a suitable justified method of analysis.

On the basis of a study of the literature and experimental research the Committee reached the following conclusions with regard to the main problems:

a. In the case of good bond at the contact face (which can be achieved if the contact face is rough or has at most "off-the-form" smoothness, but is in any case properly clean) it is not necessary to provide reinforcement to resist a shear force T_d if:

$$\frac{T_d}{bh} \leqq 0{,}3 f_b$$

b. If $T_d/bh > 0{,}3f_b$, then it is necessary to provide reinforcement to resist the entire shear force. If this shear reinforcement comprises a portion ω_{t1} which is installed only in the lower component of the composite beam and a portion ω_{t2} which extends through the whole depth of the beam, the shear force capacity T_{us} of the beam as expressed by the double (lattice) truss analogy will be:

$$T_{us} = T_{u1} + T_{u2}$$

where:

$$T_{u1} = 0{,}9 b h_1 f_{et} \omega_{t1}$$

$$T_{u2} = 0{,}9 b h f_{et} \omega_{t2}$$

It is to be noted that these formulas can be validly applied only in so far as the "truss" in question can indeed develop.

From the foregoing it follows that for a given ω_{t1}:

$$\omega_{t2} \geqq \frac{1}{0{,}9 f_{et}} \frac{T_d}{bh} - \frac{h_1}{h} \omega_{t1}$$

It appears that the following requirement can be satisfied with different combinations of ω_{t1} and ω_{t2}:

$$T_{us} \geqq T_d$$

In the case of a positive bending moment it should be realized that, in consequence

of the lower effective depth of one of the trusses, the bending moment capacity M_{us} will likewise be less in the region where $T_d > T_{u2}$. In that region:

$$M_{us} = \left(\frac{T_{u2}}{T_d} + \frac{h_1}{h}\frac{T_d - T_{u2}}{T_d}\right) M_u$$

Otherwise for $T_d \leqq T_{u2}$:

$$M_{us} = M_u$$

In the case of a negative bending moment the following relationship exists:

$$M_{us} = M_{u1} + M_{u2}$$

where:

$$M_{u1} = A_{a1} f_e z_1 \leqq \frac{T_{u1}}{T_d} M_d$$

$$M_{u2} = A_{a2} f_e z \leqq \frac{T_{u2}}{T_d} M_d$$

c. In the construction stage of the beam of course only the shear force capacity and bending moment capacity of the lower component of the beam are available, and then only those stirrups can permissibly be taken into account which completely enclose both the tensile and the compressive zone of that component.
d. If there is a smooth contact face, a larger deflection than that of a corresponding monolithic structure will have to be reckoned with. On the other hand, cracking will in general not be more unfavourable than in a monolithic structure.
e. Shrinkage differences and other effects that can give rise to initial stresses in the beam should be conceived as a loading that may affect the question as to whether or not any shear reinforcement should be provided. In general, however, such stresses have no effect on the quantity of shear reinforcement. An alternating load may also be a reason for installing shear reinforcement sooner than would otherwise be considered necessary.
f. Prestressing one of the components of the composite beam and interconnection of precast concrete components in the case of continuous beams are matters which call for further investigation.

NOTATION

A_a	total cross-sectional area of main reinforcement
A_{at}	cross-sectional area of a stirrup (two-leg)
b	width of beam
c	concrete cover
$(EI)_g$	flexural stiffness of a cracked monolithic beam section
$(EI)_{gs}$	flexural stiffness of a cracked composite beam section
F	load
F_{br}	measured failure load
F_h	load at which the first horizontal crack at the interface is found to occur
F_u	theoretical failure load of a monolithic beam
F_{us}	theoretical failure load of a composite beam
f_b	design value of concrete tensile strength
f_{bm}	mean tensile strength of lowest quality concrete $= 1 + \frac{1}{20} f'_{cm}$
f'_{cm}	mean cube strength of concrete
f_e	yield point of main reinforcement
f_{et}	yield point of stirrup reinforcement
$f_{0,2}$	0.2% proof stress of main reinforcement
h	effective depth of beam section
l_t	span of a beam
M	bending moment
M_d	design value of bending moment
M_u	theoretical failure moment of a monolithic beam
M_{us}	theoretical failure moment of a composite beam
T	shear force
T_d	design value of shear force
T_u	theoretical failure shear force of a monolithic beam
T_{us}	theoretical failure shear force of a composite beam
t	stirrup spacing
z	internal lever arm associated with failure moment M_u
γ	coefficient associated with the limit value considered
δ	deflection of beam at mid-span
δ_v	horizontal deformation (lengthening or shortening) at the interface, for each gauge length
η	coefficient of co-operation
σ_{br}	tensile stress due to shrinkage
τ	nominal shear stress $= T/bh$
τ_u	nominal failure shear stress $= T_u/bh$
τ_{uz}	failure shear stress $= T_u/bz$
ω	main reinforcement fraction $= \omega_0/100 = A_a/bh$

ω_0 geometric reinforcement percentage of a rectangular section referred to the effective depth $= 100A_a/bh$

ω_t stirrup reinforcement fraction actually present $= A_a/bt$

ω_{tv} requisite stirrup reinforcement fraction

If a symbol is provided with a subscript 1, the quantity represented by it is considered in so far as it performs a function only in the lower component. The subscript 2 indicates that the quantity in question is to be considered in so far as it performs a function only in the pattern of forces acting through the full depth of the beam.

Composite beams

1 Introduction

Although the term "composite structures" can be taken to comprise a large number of structural types, it is usually confined to members which are composed of two (or more) components. Familiar examples are laminated timber frames, steel-concrete beams or slabs, and concrete-concrete beams or slabs. The present report is concerned only with the behaviour of beams or slabs comprising two concrete components. This structural type, embodying the combination of precast concrete and in-situ concrete, is increasingly being used in building construction.

A composite concrete structure can be schematized as shown in Fig. 1. The specific problems arising in connection with a structure of this kind are bound up with the presence of a contact surface, or interface, between the two components, where forces

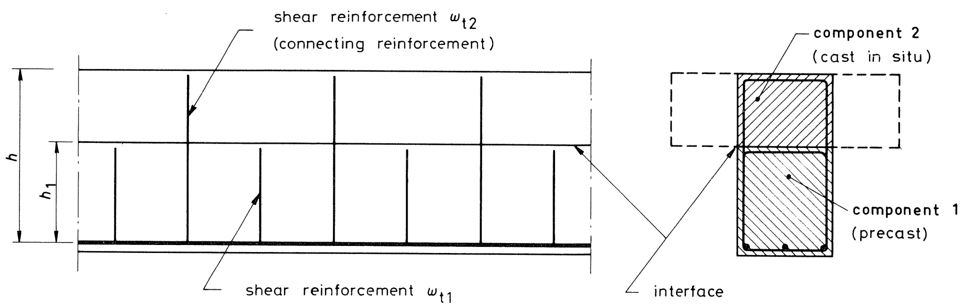

Fig. 1. Diagram of a composite beam.

have to be transmitted from one component to the other. This transmission has to be achieved in some appropriate manner through concrete-to-concrete bond or, in the absence of bond, by means of suitably chosen reinforcement and possible interfacial friction.

As there was still some lack of knowledge concerning the manner in which good co-operation between the two components of a composite concrete structure can be ensured, the CUR Committee C 19 "Co-operation of precast beams with in-situ concrete", which had been set up for the purpose, was asked to investigate the problem. The result of that research comprising a study of the literature as well as experimental investigations are contained in the present report.

In the first place the report examines the principal problems confronting the designer of composite beams. The conclusions derived from the research are furthermore given here and are, as far as possible, accompanied by recommendations for practical application.

Finally, the experimental investigations and the results thereof are briefly reviewed in Appendix A.

2 Influence of the interface

2.1 *Bond and tensile strength of the concrete at the interface*

The presence of an interface, i.e. a boundary surface or surface of contact, causes to some extent a local weakening of the material, so that a preferred direction for possible cracking is introduced. In the case of a beam or slab such weakening may cause horizontal cracking and, possibly, horizontal displacement. Obviously, good bond (of the one concrete to the other) is important in deferring for as long as possible the occurence of cracking at the interface. It is, in this context, necessary to consider whether and, if so, to what extent the bond that develops is dependent on the roughness of the interface.* A rough interface can indeed be expected to have a favourable effect upon the stiffness of the beam after horizontal cracks have formed, since roughness must increase the resistance to horizontal displacement.

Although it is in principle possible to carry out a standard test for determining the strength of a joint between older and younger concrete (i.e., where fresh concrete has been cast against concrete that has already hardened) by means of, for example, a splitting test on a cube specimen containing such a joint, a test of this kind would not be very meaningful with regard to the interface envisaged here. This is so because the pattern of stress at the interface in a composite beam is fairly complex and quite different from that in a cube under splitting test load. It is therefore better to base oneself on the results of tests performed on composite beams.

As appears from the test results published in the literature, it can be assumed that the strength of a composite beam comprising a (practically) rough interface is equal to the strength of a comparable monolithic beam. The effect of any connecting reinforcement across the interface is ignored in this assumption. The available published data as to the strength of a smooth interface are insufficient to permit clear-cut conclusions with regard to a safe lower limit to be adopted for the ultimate shear stress.

In the experimental research described in Appendix A it was found that even in the case of a smooth interface (off-the-form smoothness) so much bond could still develop as to enable the composite beam to attain the same strength as that of an otherwise identical monolithic beam. However, on account of the uncertainty that may still exist with regard to the bond at the interface in the event of rather unfavourable practical conditions, it is nevertheless advisable to establish a limiting value for the tensile strength of the concrete at the interface. Above this limiting value it will be necessary to provide shear reinforcement, on the understanding that the entire shear force (both at the interface and in the two components) will then have to be resisted by such reinforcement.

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* It is to be noted that, in so far as bond is concerned, a rough but dirty interface could conceivably be equivalent to a smooth but clean one.

Basing oneself on the test results, it would appear justified, for the purpose of practical calculation, to fix this lower limiting value at $0.3f_b$, subject to the following conditions:

a. the interface should be sufficiently clean to achieve optimum bond between the two components;
b. in view of the above-mentioned uncertainty concerning the bond, the lower component of the beam should be able alone to support the total load with a reasonable factor of safety (e.g. $\gamma = 1.1$).

This implies that for a nominal shear stress

$$\frac{T_d}{bh} \leqq 0{,}3 f_b$$

a composite beam need not be reinforced for shear.

2.2 *Position of the interface in the cross-section*

Cracking (destruction of bond) at the interface is liable to occur sooner if the shear cracks caused by diagonal tensile stresses reach the interface. For this reason a more unfavourable result is to be expected if the interface is situated in the tensile zone than if it is situated in the compressive zone of the beam. However, this distinction is not important in a case where

$$\frac{T_d}{bh} > 0{,}3 f_b$$

(i.e., where cracking is to be expected) and the total shear force is resisted by reinforcement.

2.3 *Difference in concrete quality between the two components*

In general, the two concrete components of the beam will differ in quality. The design value of the concrete tensile strength f_b should therefore logically be based on the lower quality.

As regards the compressive strength it is to be noted that the compressive zone of the concrete may be located in one or in both components and that, accordingly, one or both concrete qualities will have to be taken into account. This matter will be further considered in Chapter 3.

3 Design of the shear reinforcement

3.1 *Truss analogy*

It might be supposed that the stirrups which pass through the interface could be designed not only to resist the tensile force associated with the conventional shear analysis ("diagonal" tensile stresses, truss analogy), but also to resist a shear force acting at the interface. This shear force is due to the lattice struts (diagonal compression members) intersected at the level of the interface, as indicated in Fig. 2.

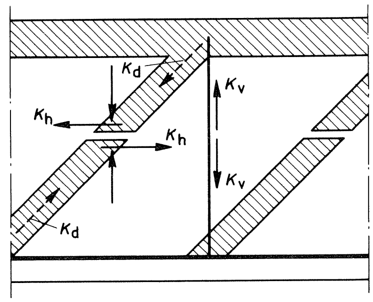

Fig. 2. Truss analogy, applied to a beam with diagonal compression members cut horizontally.

From the test results reported in the literature and also from our own research it emerges, however, that this additional shear force need not be taken into account, so that the stirrups in question can be designed in the usual way. This means that if all the shear reinforcement is present through the full depth of the beam (i.e., in both components) and if moreover the total shear force is resisted by this reinforcement, the requisite quantity of this reinforcement can permissinly be designed in accordance with the method customarily applied in the design of monolithic (non-composite) beams.

A different situation will exist, however, if not all the shear reinforcement extends through the full depth of the beam, as is liable to occur in composite beams.

Tests have shown that the shear strength (capacity to resist shear force) of a simply-supported beam comprising two (reinforced) concrete components can be approximated by the formula:

$$T_{us} = T_{u1} + T_{u2}$$

where:

T_{us} = shear strength of the composite beam;

T_{u1} = shear strength based on the stirrup reinforcement fraction ω_{t1} present only in the lower component and on the associated effective depth h_1; where $\omega_{t1} = A_{at1}/bt_1$;

T_{u2} = shear strength based on the stirrup reinforcement fraction ω_{t2} extending through the full depth of the beam and on the associated effective depth h; where $\omega_{t2} = A_{at2}/bt_2$.

In other words, a composite beam can be designed on the basis of the usual (lattice) truss analogy adopted for monolithic beams, except that now there are two interpenetrating truss systems to be considered, with effective depths h_1 and h respectively (see Fig. 3).

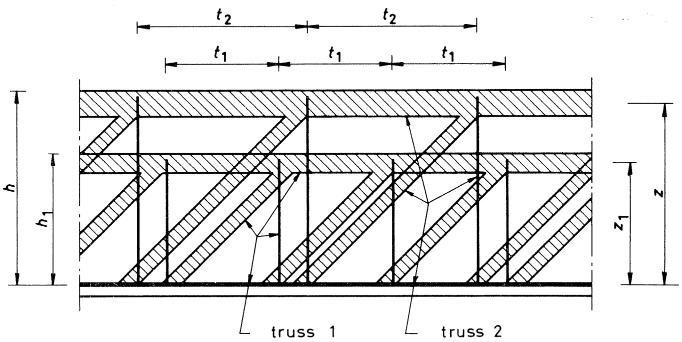

Fig. 3. Double truss analogy.

Now if it is assumed that the sloping struts (diagonal compressive members) of the truss form an angle of 45° with the horizontal members, the shear strength will, if vertical stirrups are installed, be expressed by:

$$T_u = 0{,}9bhf_{et}\omega_t$$

so that also:

$$T_{u1} = 0{,}9bh_1f_{et}\omega_{t1} \tag{2}$$

$$T_{u2} = 0{,}9bhf_{et}\omega_{t2} \tag{3}$$

whence:

$$T_{us} = 0{,}9bhf_{et}\left(\frac{h_1}{h}\omega_{t1} + \omega_{t2}\right) \tag{4}$$

As appears from this, the shear strength of such a beam is less than in the case where all the shear reinforcement $\omega_t = \omega_{t1} + \omega_{t2}$ extends through the full depth h.

Of course, in analysing a composite beam on the basis of this kind of approximation, the calculation of its strength in bending (capacity to resist bending moment) should likewise be based on the double truss analogy. Two cases may occur in connection with this.

If the design value of the shear force T_d occuring at a particular section is less than the shear strength T_{u2} alone, the failure moment (ultimate moment) M_{us} of the composite beam at that section will be equal to the failure moment M_u of the corresponding monolithic beam; therefore:

$$T_d \leqq T_{u2} \quad \rightarrow \quad M_{us} = M_u$$

Where the shear force T_d in a composite beam is larger than T_{u2} (but obviously smaller than T_{us}) it can be assumed that the load corresponding to the shear force T_{u2} is resisted by truss 2, while the rest of the shear force $(T_d - T_{u2})$ has to be resisted by truss 1. Therefore in general (in the case of a positive bending moment):

$$T_{us} \geqq T_d > T_{u2} \quad \rightarrow \quad M_{us} = \left(\frac{T_{u2}}{T_d} + \frac{h_1}{h}\frac{T_d - T_{u2}}{T_d}\right) M_u \tag{5}$$

The contribution that truss 2 can make follows from:

$$\frac{M_{u2}}{M_u} = \frac{T_{u2}}{T_d}$$

so that:

$$M_{u2} = \frac{T_{u2}}{T_d} M_u$$

while the share that remains for truss 1 is:

$$\frac{M_{u1}}{\frac{h_1}{h} M_u} = \frac{T_d - T_{u2}}{T_d}$$

so that:

$$M_{u1} = \frac{T_d - T_{u2}}{T_d} \frac{h_1}{h} M_u$$

3.2 *Construction stage*

In the present context the "construction stage" denotes the situation in which only the lower component has to be capable of supporting the load acting upon, i.e., up to the point of time when the in-situ concrete component has attained the required strength. In this stage the shear strength can be determined from the formula already given, namely:

$$T_{u1} = 0{,}9 b h_1 f_{et} \omega_{t1} \tag{2}$$

or, rewritten in the form of a design formula:

$$\omega_{t1} \geqq \frac{T_{d(\text{constr. stage})}}{0{,}9 b h_1 f_{et}} \tag{6}$$

This presupposes that the projecting stirrup (ω_{12}) are not sufficiently well anchored in the compressive zone of the concrete, so that they are not allowed to be taken into account.

However, if these projecting stirrups are welded to the longitudinal reinforcing bars in the compressive zone (structural welds), in which case the anchorage requirement can be presumed to be fulfilled, then these stirrups, too, are allowed to be taken into account. Only then does the following relationship hold:

$$T_{u1} = 0{,}9bh_1 f_{et}(\omega_{t1} + \omega_{t2}) \qquad (7)$$

It is to be noted that in the construction stage, besides the dead weight of the two components of the beam, other loads associated with the construction procedure may also be acting, e.g., stored materials, handling appliance, etc. The lower component must be designed to support all these loads, in which case a lower value for γ can customarily be adopted ($\gamma = 1.4$). This condition can be satisfied in various ways, the combination of ω_{t1}, ω_{t2}, h_1 and temporary supports (if any) being dependent on economic considerations and on the desired loadbearing capacity of the beam in the service stage, i.e., under working load.

3.3 *Service stage*

In principle, the procedure for the analysis of a composite beam has already explained in 3.1.

Assuming the first component to contain reinforcement ω_{t1} which is able to resist the shear force T_{u1} as expressed by formula (2) and furthermore assuming $T_d/bh > 0.3f_b$, it will be necessary to provide stirrup reinforcement ω_{t2} for resisting the portion $T_d - T_{u1}$.

From:

$$T_{u2} \geqq T_d - T_{u1}$$

and the formulas (2) and (3) it follows that:

$$\omega_{t2} \geqq \frac{T_d - T_{u1}}{0{,}9bhf_{et}} = \frac{1}{0{,}9f_{et}}\frac{T_d}{bh} - \frac{h_1}{h}\omega_{t1} \qquad (8)$$

In fact this constitutes a design method in which the shear stresses at the interface due to the "shear force" $T_d - T_{u1}$ determine the quantity of connecting reinforcement. It is to be noted, however, that such an approach to the problem could lead to the conclusion that the shear forces in question can adequately be resisted by means of dowel-type shear connectors (i.e., relatively short pegs). From the foregoing considerations it emerges, however, that it is essential to give so-called connecting reinforcement the form of the usual shear reinforcement.

In 3.1 it has already been stated that installing a reinforcement fraction ω_{t1} with the limited depth h_1 may result in a reduction of the strength in bending, i.e., the failure moment. It should accordingly be verified with the aid of formula (5) wether, at the section under consideration, the condition $M_{us} \geqq M_d$ is satisfied and, if necessary, a different combination of ω_{t1} and ω_{t2} should be chosen.

It is evident that the assumption of the double truss analogy is justified only if such a pattern of forces is indeed able to develop in the ultimate stage, i.e., at failure of the beam. Tests have shown the formulas given here to be justified, so that such a pattern of forces can be presumed to develop in a beam subject to a positive bending moment and comprising an interface that is not located extremely high up or extremely low down.

If there is a positive bending moment (main reinforcement in the lower component) and the interface is located extremely high up in the beam, the concrete compresive zones in the construction stage and in the service stage will partly coincide, so that in principle it will, in analysing the sections of the composite beam, be necessary to take account of the strains or the concrete stresses which have already been introduced in the constuction stage. Since the compressive stresses in the concrete are only of minor influence on the failure moment when the usual percentages of (main) reinforcement are provided, the superposition of the two sets of strains will in most cases not cause any appreciable reduction in failure moment.

In the foregoing considerations it has always been presupposed that a positive bending moment occurs at the section where the shear reinforcement is analysed, so that in the service stage there are – in so far as the assumed trusses are concerned – one tie (i.e., tensile member, constituted by the main reinforcement) and two struts (i.e., compression members, constituted by the compressive zones of the concrete).

An entirely different situation exists at a support where a negative bending moment occurs. Presupposing the structure is so designed that the lower component has to be capable of resisting a negative moment at the support also in the construction stage – for which purpose an appropriate longitudinal reinforcement is provided (see Fig. 4) – the analysis in the service stage will have to be based on two trusses with two separate tensile chords (two layers of main reinforcement) and one combined compressive chord (the concrete compressive zone).

A safe approximation will of course be obtained by neglecting the main tensile reinforcement and the stirrups which are installed only in the lower component and by basing the analysis merely on the main reinforcement installed in the upper component and on the stirrups (ω_{t2} in Fig. 4) which extend through both components. In a case where the interface is located high up in the beam, however, the designer will

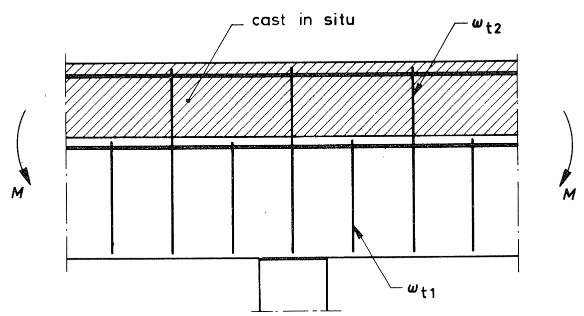

Fig. 4. Possible reinforcement at a support.

generally try to utilize the reinforcement in both components. In either case it must be borne in mind that the compressive zone of the concrete has already undergone deformation in the construction stage and that this deformation should also be taken into account in the analysis of the final beam.

For determining the shear strength T_{us}, or the associated reinforcement fractions ω_{t1} and ω_{t2}, the formulas (1) to (8) may be used in this case too, with the exception of formula (5), which now cannot be employed for calculating the failure moment, the reason being that there are two separate tensile chords, for which the concept of M_{us} as earlier defined is not valid.

The failure moment M_{us} can now be expressed as the sum of the failure moments of the two separate trusses:

$$M_{us} = M_{u1} + M_{u2} \tag{9}$$

These failure moments are of course dependent on the quantities of main reinforcement A_{a1} and A_{a2} provided, but may moreover be limited by the shear strength of the trusses. Hence the following values may be taken into account for M_{u1} and M_{u2}:

$$M_{u1} = A_{a1} f_e z_1 \leqq \frac{T_{u1}}{T_d} M_d \tag{10}$$

$$M_{u2} = A_{a2} f_e z \leqq \frac{T_{u2}}{T_d} M_d \tag{11}$$

With the aid of these formulas the flexure-resisting reinforcement which satisfies the following condition can be calculated:

$$M_{us} = M_{u1} + M_{u2} \geqq M_d$$

For example, if the shear reinforcement of both trusses and the flexure-resisting reinforcement of the first truss (i.e., the shear strengths T_{u1} and T_{u2} and the failure moment M_{u1}) have already been determined, the requisite flexure-resisting reinforcement in the second truss is obtained from:

$$\omega_2 = \frac{A_{a2}}{bh} \geqq \frac{M_d - M_{u1}}{bhf_e z} \approx \frac{M_d - M_{u1}}{0{,}9bh^2 f_e}$$

The assumption of two separate trusses with different depths entails a reduction both of the failure moment (i.e., the capacity of the beam to resist bending) and of the shear strength (i.e., its capacity to resist shear force) in comparison with a monolithic beam. In general, therefore, it is recommendable to have the interface high up in the beam or, preferably, a shear reinforcement extending through the full depth of the beam. This is more particularly relevant to T-beams in which the interface is located directly below the flange: extending the stirrups to the full depth involves only a small increase in the quantity of steel, while the shear strength is considerably increased.

At the same time, however, it should be realized that in the construction stage the projecting stirrups serve a useful purpose only if they are properly anchored below the interface (in the compressive zone or, in the case of a negative bending moment, to the flexure-resisting reinforcement installed there). Effective anchorage can be obtained, for example, by welding the stirrups to the longitudinal bars (using structural welds). In a case where at least twice as much stirrup steel extends through the interface than is needed for resisting the shear force in the construction stage it may not be necessary to take any special precautions for ensuring good anchorage, but it is not possible to be sure of this.

3.4 *Deflection*

For calculation of the deflection it is necessary to know the flexural stiffness of the beam. If first a composite beam is considered which has a failure load equal to that of the corresponding monolithic beam, then $M_{us} = M_u$ and the requisite shear reinforcement will be present through the full depth. The flexural stiffness $(EI)_{gs}$ of the cracked composite beam will to a great extent be dependent on the resistance to shearing along the interface. This can be approximately expressed by the formula:

$$(EI)_{gs} = (EI)_g \left\{ \eta + (1-\eta) \left(\frac{h_1}{h} \right)^2 \right\}$$

In this expression $(EI)_g$ denotes the flexural stiffness of the corresponding monolithic beam and η the co-operation coefficient based on the above-mentioned resistance to shearing. For a very smooth interface, without bond but provided with connecting reinforcement, a value of $\eta = 0{,}5$ can be adopted. For a monolithic beam this coefficient is $\eta = 1{,}0$.

If the failure load of the composite beam is smaller than that of the monolithic beam – i.e., $M_{us} < M_u$ –then, as tests have shown, the flexural stiffness of the composite beam is proportional to M_{us}, so that the following expression is obtained:

$$(EI)_{gs} = (EI)_g \left\{ \eta + (1-\eta) \left(\frac{h_1}{h} \right)^2 \right\} \frac{M_{us}}{M_u} \qquad (12)$$

It is recommended that the value 0,5 be adopted for η, even in the case of a smooth interface with (initial) bond, since horizontal cracking may certainly occur in such a case.

If the interface is roughened, it may be pressumed that sufficient shearing resistance will be available, even after possible destruction of the bond, to ensure that the stiffness will be equal to that of the corresponding monolithic beam.*

* Although it is not possible to give an accurate definition of "smooth" and "rough", a "smooth surface" may be taken to mean an (as-placed) concrete surface which corresponds to an off-the-form (or smoother) face, while a "rough surface" may mean an (as-placed) concrete surface which is, or has been made, so rough that distinctly visible and evenly distributed irregularities are present on it.

3.5 *Cracking*

As regards cracking, the situation in a composite beam can be expected to be no more unfavourable than that in a monolithic one. In general, it is assumed that the crack width is proportional to the steel stress and to the spacing of the cracks and that they are more closely spaced according as the (main) reinforcement percentage is higher.

Now in the case of a simply-supported composite beam in which full cracking has developed already in the construction stage the crack spacing (distance between adjacent cracks) can, on account of the relatively high reinforcement percentage in that stage ($\omega = A_a/bh_1$), only be less than in the case where cracking occurs in the service stage ($\omega = A_a/bh$). Hence it is, in general, correct to calculate the crack width as though for the identical monolithic structure with its associated crack spacing and the steel stresses that can be expected to occur in the service stage.

4 Other aspects

The problems arising in connection with the presence of an interface and the different heights of the shear reinforcement ω_{t1} and ω_{t2} have been dealt with in Chapters 2 and 3.

Other aspects that may occur with reference to composite beams are:

- inclined shear reinforcement;
- possible shrinkage, creep and temperature effects;
- possible effects of alternating load on the strength of the interface;
- prestress, if any;
- interconnection of the precast concrete components at the support.

The effect of inclined shear reinforcement, of differential shrinkage and of alternating load has been approximately investigated in a few tests. On the evidence of the results thereof and/or on the basis of the pattern of forces that can reasonably be expected to occur in the composite beam with regard to the above-mentioned factors, the following inferences can be drawn.

4.1 *Inclined shear reinforcement*

The results of tests with inclined stirrups (welded to the longitudinal reinforcement) are somewhat more favourable, both as regards failure load and as regards deflection, than those of tests with vertical stirrups. The difference is not great, however. Besides, for practical reasons, inclined stirrups will not generally be employed, the more so as there are doubts as to the effectiveness of the transmission of force between such stirrups and the longitudinal reinforcement if no welds are applied at the junctions. Inclined bars serving as shear reinforcement will therefore in general be confined to bent-up bars of the longitudinal reinforcement. As already explained with regard to vertical stirrups, these inclined bars can be taken into account by means of an analysis based on the truss analogy.

4.2 *Shrinkage, creep and temperature effects*

Since the components of a composite beam differ in quality, age and loading, stress may develop in the beam because of the fact that the components cannot freely undergo deformation independently of each other. A similar restraint effect may arise in consequence of temperature differences. The tensile stresses that are thus produced in the concrete can, just as those due to external loading, give rise to cracking. Hence the tensile stresses from both these causes should – in so far as they occur simultaneously – be taken into account in determining the magnitude of the shear force for which it becomes necessary to provide shear reinforcement. So in the case of a tensile stress due to shrinkage (σ_{br}) it is not necessary to employ shear reinforcement if:

$$\frac{T_d}{bh} + \sigma_{br} \leqq 0{,}3 f_b \tag{13}$$

It is, however, advisable not to take account of shrinkage if the shrinkage stresses are deductable from the stresses due to external loading in a case where the shrinkage value to be allowed for is only approximately known.

In other cases it will have to be judiciously decided to what extent the shrinkage stresses should be taken into account in the manner indicated here. The same applies to differential creep (if any) and to the effect of temperature differences. All three cases can be regarded as constituting a loading and be taken into account if the stresses in question are of importance as causes of cracking.

As already stated, the shrinkage stresses are due to restrained deformations. If they give rise to cracking, the restraint in general will largely be removed in consequence thereof, while according to the design method given here the total shear force is resisted by the reinforcement. For this reason the secondary stresses may make it necessary to reinforce beam even if the shear force is less than $T = 0{,}3\ f_b bh$, but in general it is not necessary to take account of those stresses in determining the *quantity* of reinforcement.

4.3 *Alternating load*

In the tests in which a composite beam was subjected to cycles of alternating load, these were found not te have an adverse effect on its strength. This does not alter the fact that the permanance of the bond at the interface in a beam subjected to prolonged and substantially alternating load (machinery foundations, etc.) is open to doubt. In such cases it is advisable to provide reinforcement for resisting the entire shear force even if $T_d/bh < 0{,}3 f_b$.

4.4 *Prestress*

No tests were performed with composite beams comprising one prestressed component.

Although the prestress and the resulting creep can be regarded as a load and be treated in accordance with the truss analogy already discussed, it is not directly evident how such a beam will behave in general, and it is difficult to indicate a limiting value of T_d/bh below which no shear reinforcement need be provided. Further investigation of the problem is desirable.

4.5 *Interconnection of precast components*

The precast concrete components of composite beams are often laid end-to-end as simply-supported beams, no rigid flexural connection being established over the intermediate supports (see Fig. 5). If a bending moment will have to be transmitted across the support in the service stage, main reinforcement will obviously have to be installed in the in-situ concrete component. Such reinforcement will have to resist the tensile force due to the negative support moment and be adequately anchored at its ends. The joint between the two precast component will moreover have to be carefully and tightly packed with concrete that can develop the requisite compressive zone.

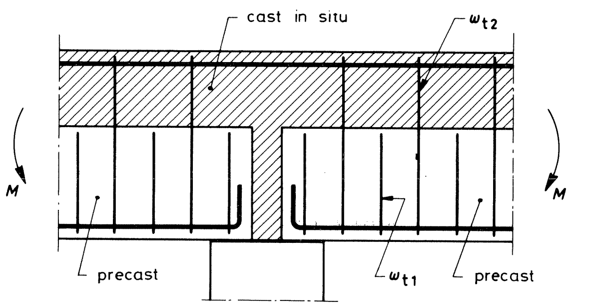

Fig. 5. Connection of the precast components at a support.

Finally, shear reinforcement (ω_{t2}) will be needed for resisting the relatively large shear forces acting on each side of the support. In principle, these arrangements will have complied with the conditions for applying the method of determining the shear reinforcement described in Chapter 3. There are, however, some uncertainties with regard to a structural detail of this kind, more particularly:

- the requirements applicable to the stirrup reinforcement if it is anchored in a prestressed precast component which does not require stirrup reinforcement;
- the condition, stated in 2.1, that if there is no shear reinforcement across the interface the lower component must by itself be able to carry the total load with some

margin of safety; that condition would necessitate either providing a support reinforcement (i.e., in the vicinity of the bearings) in the lower component or installing such mid-span reinforcement as to ensure that this component can carry the total load as a simply-supported beam;

- other possibilities of transmitting the tensile force in the main reinforcement to the compressive zone without making use of stirrups (anchorage of the main reinforcement in the compressive zone, helical reinforcement, etc.).

Of course, the analysis can in this case, too, be based on the truss analogy. However, in order to find out under what conditions the assumption as to the pattern of forces is indeed fulfilled, it is desirable to carry out further research on the above-mentioned points.

5 Examples of calculations

Two worked examples relating to a continuous composite beam are given here in order to clarify the practical application of the method of analysis proposed in this report.

The first example relates to the case where the reinforcement for resisting the negative moment over the support is installed only in the in-situ concrete. In the second example reinforcement over the support is provided both in the in-situ and in the precast concrete.

5.1 *Example 1*

Determine the quantity of reinforcement required for a composite beam consisting of a precast reinforced concrete component over which an in-situ concrete floor slab is laid (see Fig. 6). In the service stage a negative bending moment will have to be resisted at intermediate supports. In this example the reinforcement for resisting this moment is assumed to be installed only in the in- situ concrete.

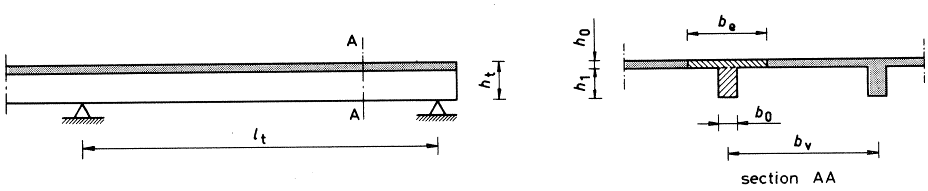

Fig. 6. Composite beam considered in Example 1.

Further data:

dimensions:	$l_t = 10$ m	$b_0 = 0{,}35$ m
	$h_t = 0{,}85$ m ($\approx \frac{1}{12} l_t$)	$b_v = 4{,}00$ m
	$h_0 = 0{,}15$ m	$b_e = 1{,}95$ m (effective width)
	$h_1 = 0{,}70$ m	$c = 25$ mm (concrete cover)

loads:	dead weight 24 kN/m^2 variable load in construction stage 0,5 kN/m^2 variable load in service stage 12 kN/m^2
steel grades:	main reinforcement FeB 400 HK with $f_{0,2} = 400$ N/mm^2 stirrups FeB 400 HW with $f_{et} = 400$ N/mm^2
concrete quality classes:	beam B 37,5 with $f_b' = 30$ N/mm^2 and $f_b = 1{,}8$ N/mm^2 floor B 17,5 with $f_b' = 14$ N/mm^2 and $f_b = 1{,}2$ N/mm^2

The calculations are based on the bending moment and shear diagrams indicated in Fig. 7. It is to be noted that p is the design value of the total uniformly distributed load in the service stage. It is furthermore assumed that in the ultimate stage, i.e., at failure, a certain amount of moment redistribution will have occured, so that both the mid-span and the support moment can be taken as equal to $\frac{1}{12}pl_t^2$. This is a fairly arbitrary assumption. In practical cases the designer will have to make a justified choice.

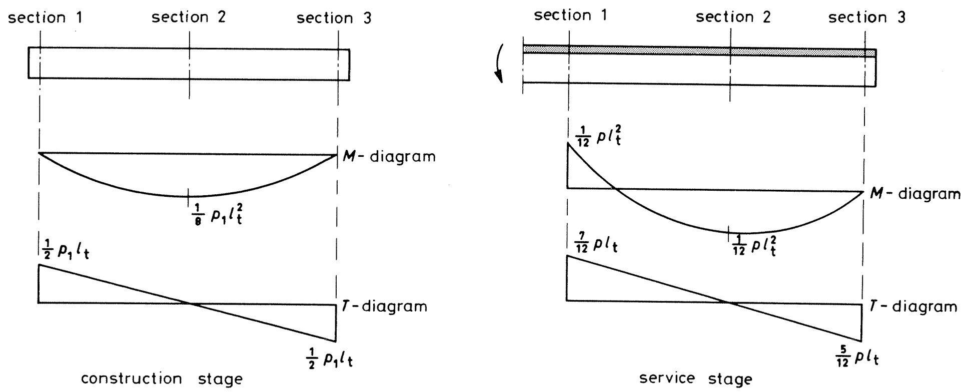

Fig. 7. Bending moment and shear force diagrams in the construction stage and the service stage.

5.1.1 Construction stage

The design values of the loads are:

dead weight of beam	$= 1{,}4 \cdot 0{,}35 \cdot 0{,}70 \cdot 24 =$	8,23 kN/m
dead weight of floor	$= 1{,}4 \cdot 0{,}15 \cdot 4{,}00 \cdot 24 =$	20,16 kN/m
variable load	$= 1{,}4 \cdot 4 \cdot 0{,}5 =$	2,80 kN/m
	$p_1 =$	31,19 kN/m

The design values of the mid-span moment and the shear force are:

$$M_d = \tfrac{1}{8}p_1 l_t^2 = \tfrac{1}{8} \cdot 31{,}19 \cdot 10^2 = 390 \text{ kNm}$$

$$T_d = \tfrac{1}{2}p_1 l_t = \tfrac{1}{2} \cdot 31{,}19 \cdot 10 = 156 \text{ kN}$$

Main reinforcement at section 2:

The design formula adopted for determining the required quantity of main reinforcement A_a is:

$$A_a \geqq \frac{M_d}{0{,}9h_1 f_{0,2}}$$

Estimation of h_1:

$$h_1 = 700-(25+10+15) = 650 \text{ mm}$$

Now:

$$A_a \geqq \frac{390\cdot 10^6}{0{,}9\cdot 650\cdot 400} = 1665 \text{ mm}^2$$

Shear reinforcement at sections 1 and 3:

$$\frac{T_d}{b_0 h_1} = \frac{156\cdot 10^3}{350\cdot 650} = 0{,}69 \text{ N/mm}^2$$

This value is larger than $0{,}3f_b$ $(= 0{,}3 \times 1{,}8 = 0{,}54 \text{ N/mm}^2)$, so that the entire shear force must be resisted by reinforcement.

The quantity of stirrup reinforcement, using vertical stirrups, is

$$\omega_{t1} \geqq \frac{T_d}{0{,}9b_0 h_1 f_{et}} = \frac{T_d}{b_0 h_1}\frac{1}{0{,}9f_{et}} = 0{,}69\frac{1}{0{,}9\cdot 400} = 0{,}0019$$

5.1.2 Service stage

The design values of the loads are:

dead weight of beam and floor	$= 1{,}7(8{,}23+20{,}16)/1{,}4 =$	34,48 kN/m
variable load	$= 1{,}7\cdot 4\cdot 12$ $=$	81,60 kN/m
	$p =$	116,08 kN/m

The design values of the moment and shear forces are:

$$M_d(\text{section 2}) = M_d(\text{section 1}) = \tfrac{1}{12}pl_t^2 = \tfrac{1}{12}\cdot 116{,}08\cdot 10^2 = 967 \text{ kNm}$$

$$T_d(\text{section 3}) = \tfrac{5}{12}pl_t = \tfrac{5}{12}\cdot 116{,}08\cdot 10 = 484 \text{ kN}$$

$$T_d(\text{section 1}) = \tfrac{7}{12}pl_t = \tfrac{7}{12}\cdot 116{,}08\cdot 10 = 677 \text{ kN}$$

Main reinforcement at section 2:

Estimation of h:

$$h = 850-(25+10+15) = 800 \text{ mm}$$

$$A_a \geqq \frac{M_d}{0{,}9hf_{0,2}} = \frac{967\cdot 10^6}{0{,}9\cdot 800\cdot 400} = 3360 \text{ mm}^2 \qquad (\omega_1 \geqq 0{,}00215)$$

Shear reinforcement at section 3:

$$\frac{T_d}{b_0 h} = \frac{484 \cdot 10^3}{350 \cdot 800} = 1{,}73 \text{ N/mm}^2$$

This value is larger than $0{,}3 f_b (= 0{,}3 \times 1{,}2 = 0{,}36 \text{ N/mm}^2)$, so that the entire shear force must be resisted by reinforcement.

On the assumption that for the construction stage the requirement $\omega_{t1} \geqq 0{,}0019$ is just satisfied, we have:

$$T_{u1} = T_d(\text{constr.}) = 156 \text{ kN}$$

For the stirrup reinforcement we now obtain:

$$\omega_{t2} \geqq \frac{T_d - T_{u1}}{0{,}9 b_0 h f_{et}} = \frac{(484 - 156)10^3}{0{,}9 \cdot 350 \cdot 800 \cdot 400} = 0{,}00325$$

Shear reinforcement at section 1:

Since

$$\frac{T_d}{b_0 h} > 0{,}3 f_b$$

it is necessary to provide shear reinforcement.

In this case there is a negative bending moment. However, since no reinforcement to resist bending moment over the intermediate support is provided in the precast beam, the truss is not present in this component; hence:

$$T_{u1} = 0$$

therefore:

$$T_{u2} \geqq T_d$$

or:

$$\omega_{t2} \geqq \frac{T_d}{0{,}9 b_0 h f_{et}} = \frac{677 \cdot 10^3}{0{,}9 \cdot 350 \cdot 800 \cdot 400} = 0{,}00672$$

Bending moment reinforcement at section 1:

As already stated, at the support:

$$T_{u2} \geqq T_d$$

so that:

$$M_{us} = M_{u2} (= M_u)$$

The requisite reinforcement is obtained from the condition $M_{us} \geqq M_d$; therefore:

$$A_a \geqq \frac{M_d}{0{,}9 h f_{0,2}} = \frac{967 \cdot 10^6}{0{,}9 \cdot 800 \cdot 400} = 3360 \text{ mm}^2 \qquad (\omega_2 \geqq 0{,}012)$$

The quantities of reinforcement which are found to be necessary in the construction stage and in the service stage are summarized in Fig. 8.

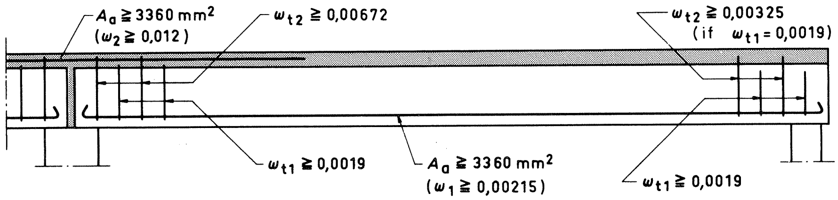

Fig. 8. Requisite quantities of reinforcement.

5.1.3 Design and check calculations

The design of the reinforcement has in principle already been done in 5.1.1 and 5.1.2. As those were only approximate calculations and as there is a certain amount of freedom in the choice of combinations of ω_1, ω_2, ω_{t1} and ω_{t2}, the following more accurate analysis is based on the reinforcement actually adopted.

Main reinforcement at section 2:

Having chosen the main reinforcement, it was investigtigated by means of an analysis (not given here) whether the requisite failure moment can indeed be developed. The following was found for the mid-span region:

$$4\varnothing 32 \text{ with } A_a = 3216 \text{ mm}^2$$

$$M_u = 996 \text{ kNm}$$

The quantity of steel therefore turns out to be a little less than the quantity originally determined. For section 2 we have $T_d = 0$, so that:

$$M_{us} = M_u = 996 \text{ kNm} (> M_d = 967 \text{ kNm})$$

Main reinforcement at section 1:

For the support the above-mentioned analysis gives:

$$4\varnothing 32 + 1\varnothing 28 \text{ with } A_a = 3832 \text{ mm}^2$$

$$M_{us} = M_u = 975 \text{ kNm} (> M_d = 967 \text{ kNm})$$

Shear reinforcement at section 3:

With (see Fig. 9)

$$\text{stirrups } \varnothing 10-225(\omega_{t1} = 0.0020) \text{ and 2 stirrups } \varnothing 10-225(\omega_{t2} = 0{,}0040)$$

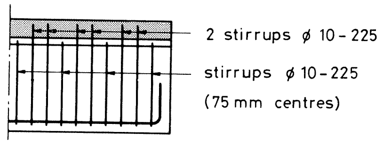

Fig. 9. Stirrup reinforcement at section 3.

the relevant conditions are satisfied (see Fig. 8). This is certainly not the only possible solution, however, for if the shear strength T_{u1} of the lower component is increased, T_{u2} may decrease. Some other solutions are shown in Fig. 10.

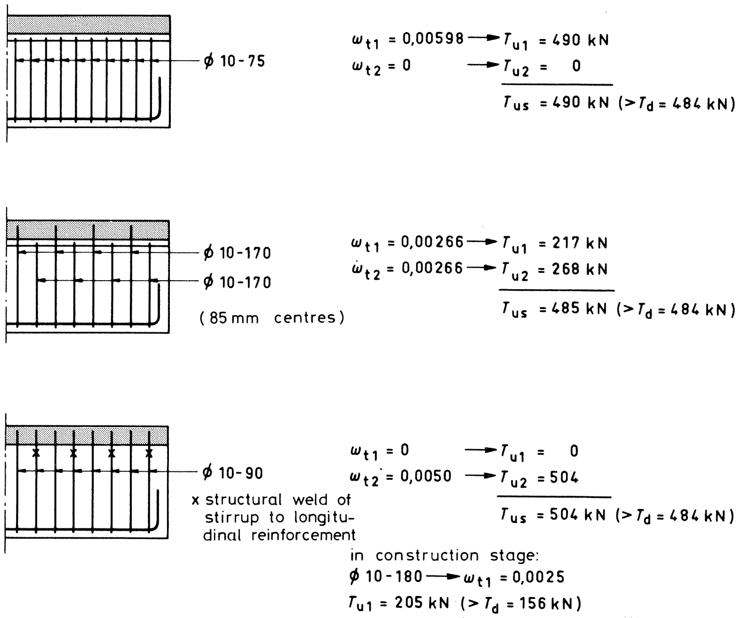

Fig. 10. Alternative solutions for the stirrup reinforcement at section 3.

Shear reinforcement at section 2:

No shear reinforcement is required if

$$T_{us} \leqq 0{,}3 f_b b_0 h = 0{,}3 \cdot 1{,}2 \cdot 350 \cdot 800 \cdot 10^{-3} = 101 \text{ kN}$$

However, since the precast beam should contain a certain minimum of stirrup reinforcement (centre-to-centre spacing max. 300 mm), these stirrups can serve also to resist shear force. For a minimum stirrup reinforcement of $\varnothing$ 10–300 through the full depth:

$$\omega_{t2}(\min) = 0{,}0015$$

$$T_{u2}(\min) = 0{,}9 b_0 h f_{et} \omega_{t2} = 0{,}9 \cdot 350 \cdot 800 \cdot 400 \cdot 0{,}0015 \cdot 10^{-3} = 151 \text{ kN}$$

The section where $T_d = 151$ kN are located at 2,87 m and 5,47 m from the right-hand support. The requisite shear reinforcement can, from the supports up to these sections, be gradually reduced in conformity with the shear force diagram along the beam. Between these sections we have $T_d < T_{u2}$, so that $M_{us} = M_u = 996$ kNm and the condition $M_{us} > M_d$ is satisfied.

Shear reinforcement at section 1:

With stirrups $\varnothing 10-65$ ($\omega_{t2} = 0{,}0069$) through the full depth the condition for the service stage is satisfied (see Fig. 8). In order to have sufficient shear strength available also in the construction stage, one out of every three stirrups is connected by means of structural welds to the longitudinal reinforcement located near the top of the precast beam. In this way good anchorage of these stirrups in the compressive zone of the precast beam is obtained (see also last paragraph of 3.3).

Hence we have for the construction stage:

$$\omega_{t1} = \tfrac{1}{3}\omega_{t2} = 0{,}0023 \quad (> \omega_{t1} = 0{,}0019)$$

It is to be noted that for the service stage, as contrasted with the situation at section 3, there is no scope for choice in distributing the shear reinforcement, because the truss in the lower component is non-existent on account of the absence of the tensile chord.

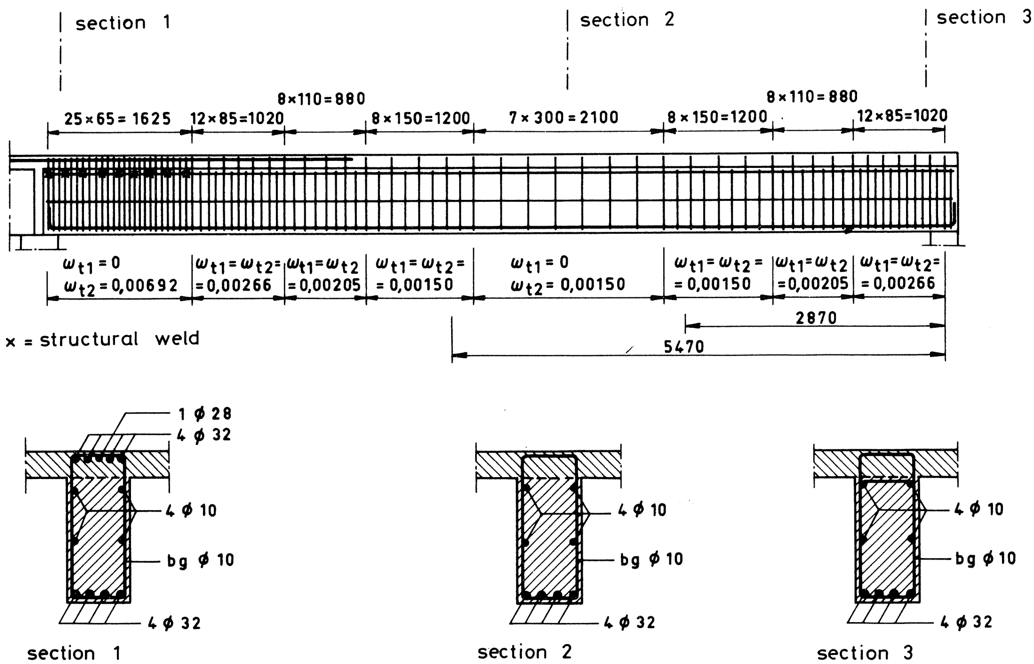

Fig. 11. Design adopted in Example 1.

5.1.4 Final design

The design finally selected is illustrated in Fig. 11. It calls for the following comments:

a. The joint between the precast beams at the intermediate support must be carefully filled with concrete whose quality is at least as good as that of the floor slab.
b. The quantity of shear reinforcement needed turns out to be fairly large. Less reinforcement will suffice if all the stirrups extend through the full depth, in which case there would moreover be only one type of stirrup. Against this, however, extra care will then have to be taken to ensure effective anchorage of these stirrups in the compressive zone of the precast beam in order to have sufficient shear strength in the construction stage.

5.2 *Example 2*

This example is concerned only with the case where the beams are so connected at an intermediate support that a bending moment reinforcement is available both in the precast and in the in-situ concrete (see Fig. 12).

The design is assumed to fulfil the requirements that a negative support moment can indeed be developed in the construction stage and that ω_{t1} is sufficient for that stage.

The given data for the service stage are:

$$\omega_{t1} = 0{,}00195 \ (\varnothing 10-230)$$
$$M_d = 700 \text{ kNm}$$
$$T_d = 490 \text{ kN}$$

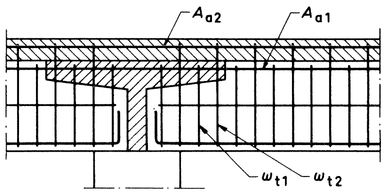

Fig. 12. Reinforcement at intermediate support.

The cross-sectional dimensions are the same as those in Example 1. The following requirements apply to the service stage:

$$M_{u1} \geqq \frac{T_{u1}}{T_d} M_d \qquad M_{u2} \geqq \frac{T_d - T_{u1}}{T_d} M_d$$

$$T_{u2} \geqq T_d - T_{u1} \qquad \omega_{t2} \geqq \frac{T_d - T_{u1}}{0{,}9 b_0 h f_{et}}$$

With:

$$T_{u1} = 0{,}9 b_0 h_1 f_{et} \omega_{t1} = 0{,}9 \cdot 350 \cdot 650 \cdot 400 \cdot 0{,}00195 \cdot 10^{-3} = 160 \text{ kN}$$

it follows that:

$$M_{u1} \geqq \frac{160}{490} 700 = 229 \text{ kNm}$$

$$M_{u2} \geqq \frac{490-160}{490} 700 = 471 \text{ kNm}$$

$$T_{u2} \geqq 490-160 = 330 \text{ kN}$$

$$\omega_{t2} \geqq \frac{330 \cdot 10^3}{0{,}9 \cdot 350 \cdot 800 \cdot 400} = 0{,}0033$$

Chosen:

2 stirrups ⌀ 10 − 230 ($\omega_{t2} = 0{,}0039$)

Estimation of the requisite support reinforcement A_{a1} and A_{a2}:

$$A_{a1} \geqq \frac{M_{u1}}{0{,}9 h_1 f_{0,2}} = \frac{229 \cdot 10^6}{0{,}9 \cdot 650 \cdot 400} = 978 \text{ mm}^2$$

$$A_{a2} \geqq \frac{M_{u2}}{0{,}9 h f_{0,2}} = \frac{471 \cdot 10^6}{0{,}9 \cdot 800 \cdot 400} = 1635 \text{ mm}^2$$

Chosen:

$A_{a1} = 1140 \text{ mm}^2$ (3 ⌀22)

$A_{a2} = 1810 \text{ mm}^2$ (4 ⌀24)

Check calculation:

For the analysis of the failure moment M_u a bilinear stress-strain ($\sigma'_b - \varepsilon'_b$) diagram is adopted. The creep coefficient to be introduced in the calculation is determined as: $\varphi_j = 2{,}1$.

Then, for concrete B 17,5:

$$E'_{bj} = \frac{E'_b}{1+\varphi_j} = \frac{25000}{1+2{,}1} = 8060 \text{ N/mm}^2$$

$$\varepsilon'_{u1} = \frac{f'_b}{E'_{bj}} 10^3 = \frac{14}{8060} 10^3 = 1{,}75‰$$

The strain and stress distribution diagrams of the section under consideration are indicated in Fig. 13:

$$N'_b = 0{,}75 b_0 x f'_b = 0{,}75 \cdot 350 \cdot x \cdot 14 \cdot 10^{-3} = 3{,}675x \text{ kN}$$

$$N_{a1} = A_{a1} f_{0,2} = 1140 \cdot 400 \cdot 10^{-3} = 456 \text{ kN, provided } \varepsilon_{a1} \geqq 2‰$$

$$N_{a2} = A_{a2} f_{0,2} = 1810 \cdot 400 \cdot 10^{-3} = 724 \text{ kN}$$

$$N'_b = N_{a1} + N_{a2}$$

$$3{,}675x = 456 + 724$$

$$x = 321 \text{ mm}$$

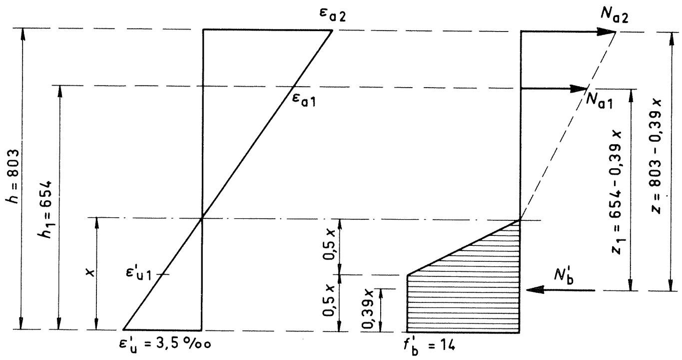

Fig. 13. Strain and stress distribution diagrams.

Check of the steel strain ε_{a1}:

$$\varepsilon_{a1} = \varepsilon'_u \frac{h_1 - x}{x} = 3{,}5 \frac{654 - 321}{321} = 3{,}6‰ \qquad (> 2‰)$$

$$z_1 = 654 - 0{,}39 \cdot 321 = 529 \text{ mm}$$

$$z = 803 - 0{,}39 \cdot 321 = 678 \text{ mm}$$

$$M_u = N_{a1} z_1 + N_{a2} z$$

$$= 456 \cdot 529 \cdot 10^{-3} + 724 \cdot 678 \cdot 10^{-3} = 732 \text{ kNm } (> M_d = 700 \text{ kNm})$$

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APPENDIX A: Experimental research

A1 Experimental arrangements

The problems associated with the design of a composite beam, as dealt with in Chapters 1 to 4, together with the conclusions drawn from a literature research [1], were the determining factors with regard to the set-up of the experimental research. A detailed description of this research has been given in [2]. Here only a brief outline will be presented.

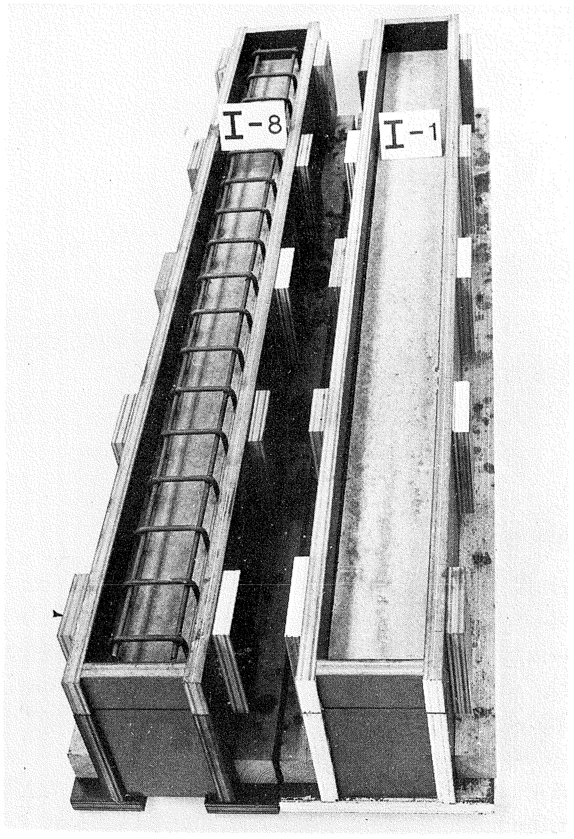


Fig. A1. Lower components of beams I-1 and I-8.

In these investigations 37 beams of rectangular cross-section were tested. There were five series of test beams, characterized as follows:

In series I there were 17 beams, which were tested with the object of investigating the effect of the location of the interface in relation to the quantity of connecting reinforcement. The beams I-1 and I-8, with the minimum and the maximum quantity of such reinforcement respectively, are shown in Fig. A1.

Series II comprised six beams in which the bond at the interface was entirely absent, this having been achieved by the interposition of a plastic sheet between the two components. The quantity of connecting reinforcement was varied, whereas the location of the interface and also the concrete quality were kept constant.

In series III four beams with stirrups inclined at 45° were tested with a view to investigating the effect of inclined connecting reinforcement. A reinforcement cage for one of the beams in this series is illustrated in Fig. A2.

In order to investigate the effect of shrinkage of the in-situ concrete portion of the beam, four beams were tested in series IV.

Finally, in series V six beams were subjected to alternating load. The quantity of connecting reinforcement and the location of the interface were varied in these tests.

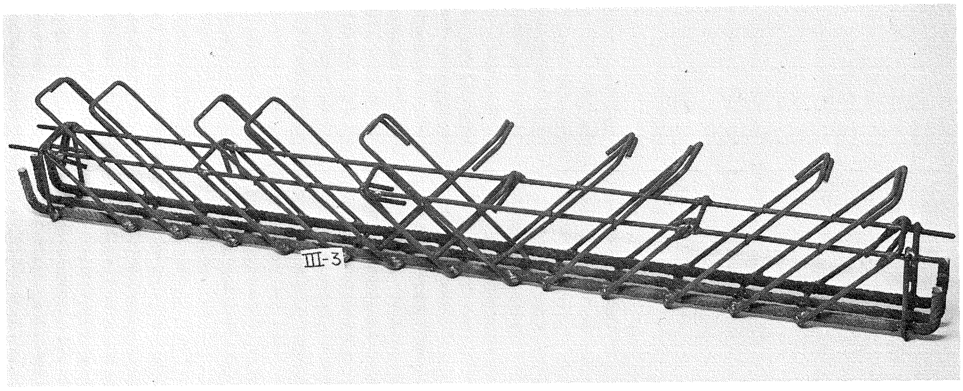


Fig. A2. Reinforcement cage of beam III-3.

A2 Testing procedures

The test beams were simply-supported on two bearings and were loaded by a point load applied at mid-span. The test set-up is shown schematically in Fig. A3.

In the static loading tests the mid-span point load was increased in increments of 10, 5 or 2,5 kN. The rate of loading was about 10 kN per minute. Besides the load, the deflection at mid-span and the strains at the gauge lengths 1 to 6 (see Fig. A3) were determined, and the cracking pattern was recorded.

In the case of the alternately loaded beams of series V the procedure employed was similar to that for the static loading tests. The alternating load cycles were applied with a frequency of 1 Hz. They were stopped when the deformation was found to undergo no appreciable further increase or when failure occured.

A photograph of the test set-up for the statically loaded beams is reproduced in Fig. A4.

A3 Test results and failure behaviour

The results of the tests are given in Table A1, together with information on the location of the interface, the average cube (compressive) strengths, the grades and

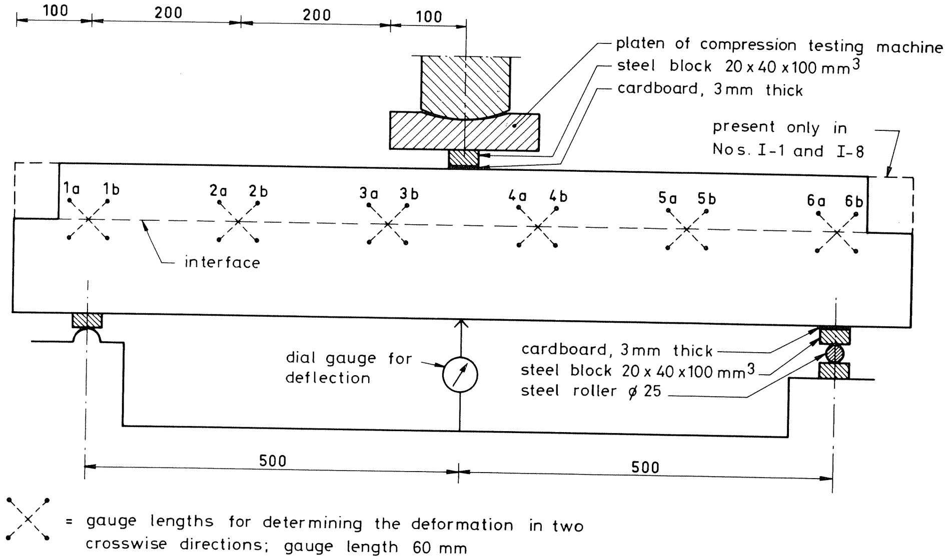


Fig. A3. Experimental set-up (schematic).

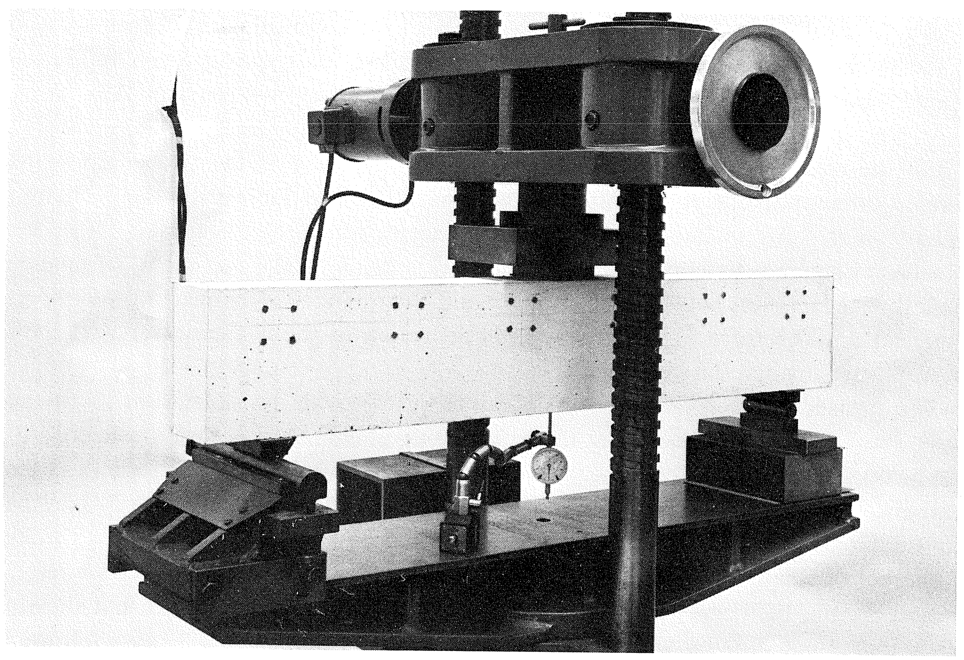


Fig. A4. Experimental set-up for static loading tests on beams.

quantities of reinforcing steel, and the calculated failure loads. The quantity of shear reinforcement required (ω_{tv}) has been determined from the shear stress at failure:

$$\tau_{uz} = \frac{T_u}{bz}$$

where:

T_u = shear force associated with the theoretical failure moment M_u

The theoretical failure load F_u is equal to $2T_u$.

The failure patterns are schematically indicated in colomn 24 of Table A1. In general it can be said that as soon as an inclined crack reaches the interface it will continue to spread along the interface, depending on the degree of bond and on the quantity of connecting reinforcement. The failure patterns fall into three main groups:

- In the first group there occurs considerable horizontal cracking, particularly if no connecting reinforcement has been provided. All the beams of series II naturally belong to this group (see Fig. A5).
- The second group is characterized by little (of the order of a few tenths of milimetres) or no horizontal cracking. Failure in this case corresponds to shear failure in a monolithic beam, this being due to the bond at the interface and/or the presence of connecting reinforcement. Striking examples of this latter condition are presented by the beams in series V (see Fig. A6).
- The beams in which bending moment failure occured form the third group. Failure of this kind is observed in the cases where both the connecting reinforcement and the shear reinforcement are approximately equal to the requisite shear reinforcement. Examples of this are Nos. I-7, I-10, I-12, III-4 and IV-4 (see Fig. A7).

From the measured deflections δ it emerges that, so long as the two components remain interconnected at the interface by bond at least at their ends, the deflection behaviour remains unchanged in the beams concerned, i.e., their flexural stiffness has not been impaired. Only if the horizontal crack extends to the end there is found to be a distinct relation between the stiffness and the quantity of connecting reinforcement, as appears from Fig. A8, where the deflection of the beams in series II have been plotted against the load F.

The measured deformations δ_v at the interface provide an indication as to the horizontal displacement occuring there. The general character is that at the ends of the beam (gauge length 1) practically no horizontal deformation occurs, where as the amount of deformation near the point of load application (gauge length 3) may be considerable. In Fig. A9 the deformation δ_v have been plotted against the load F for a number of beams in series I. At the gauge length 1 practically no horizontal defornation occurs; the curves corresponding to the gauge lengths 2 and 3 at first show a slight increase in δ_v, after which there frequently occurs an abrupt transition followed by a large increase in δ_v (beam I-11). In monolithic beams (beam I-9) this transition

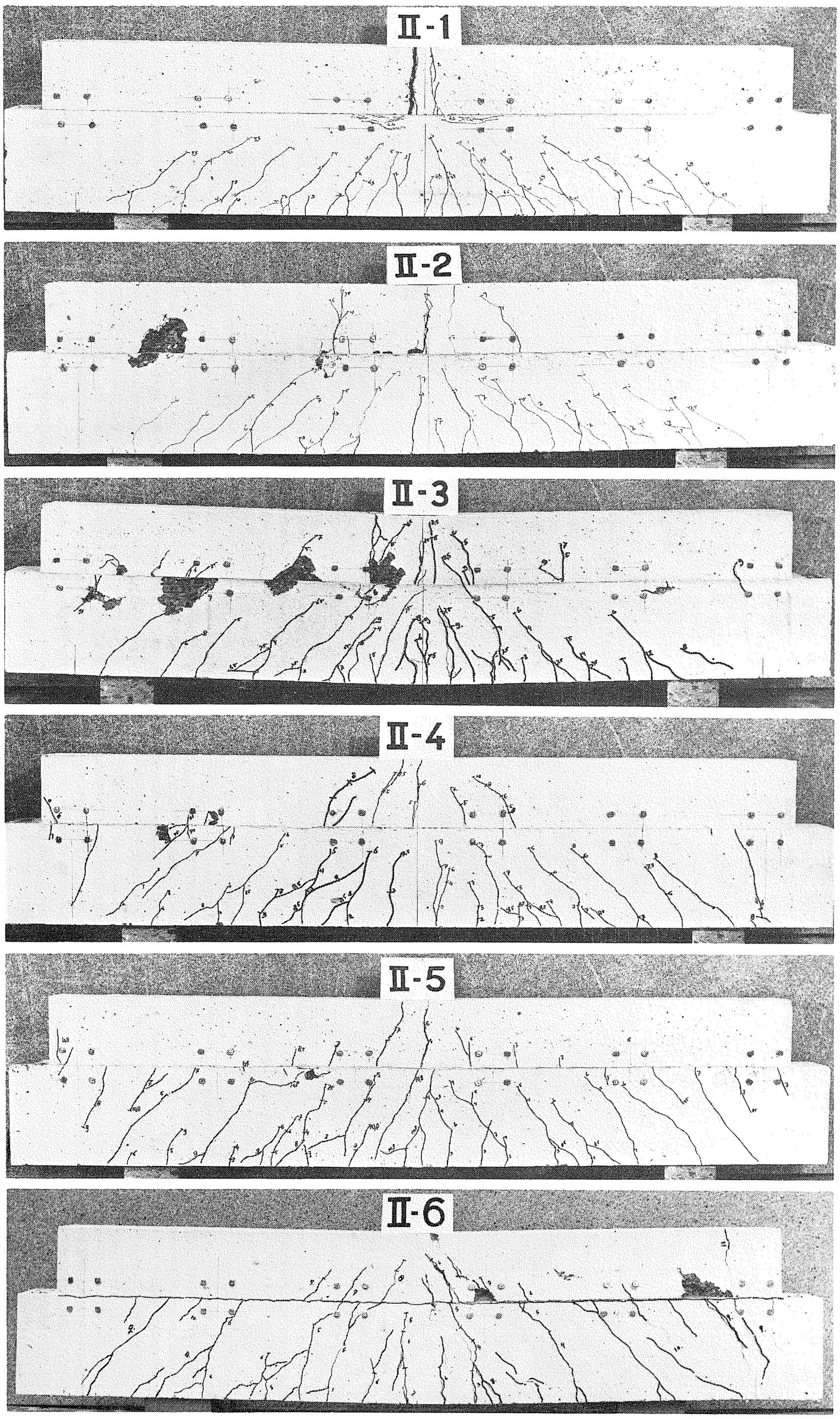


Fig. A5. Failure patterns of the beams in series II.

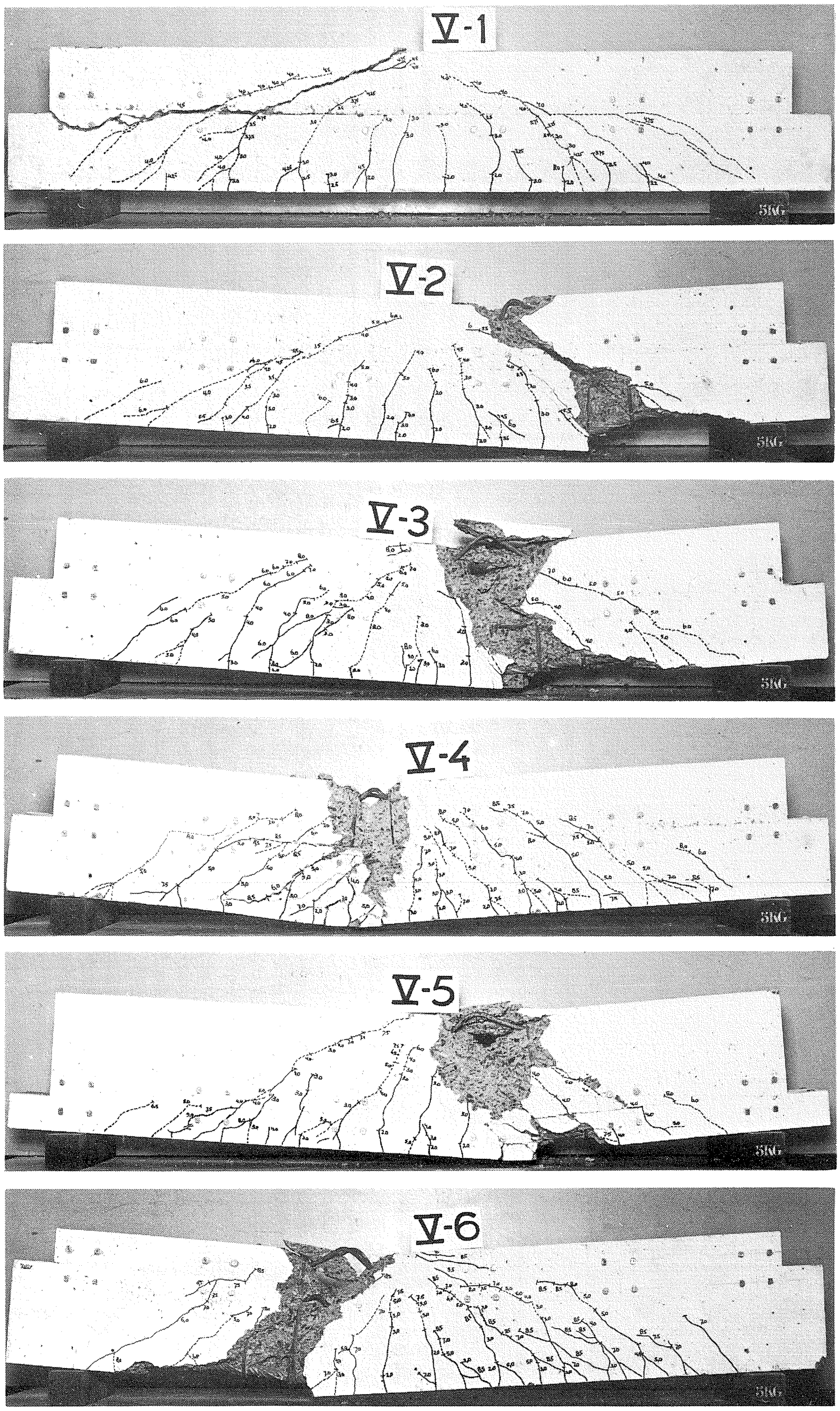


Fig. A6. Failure patterns of the beams in series V.

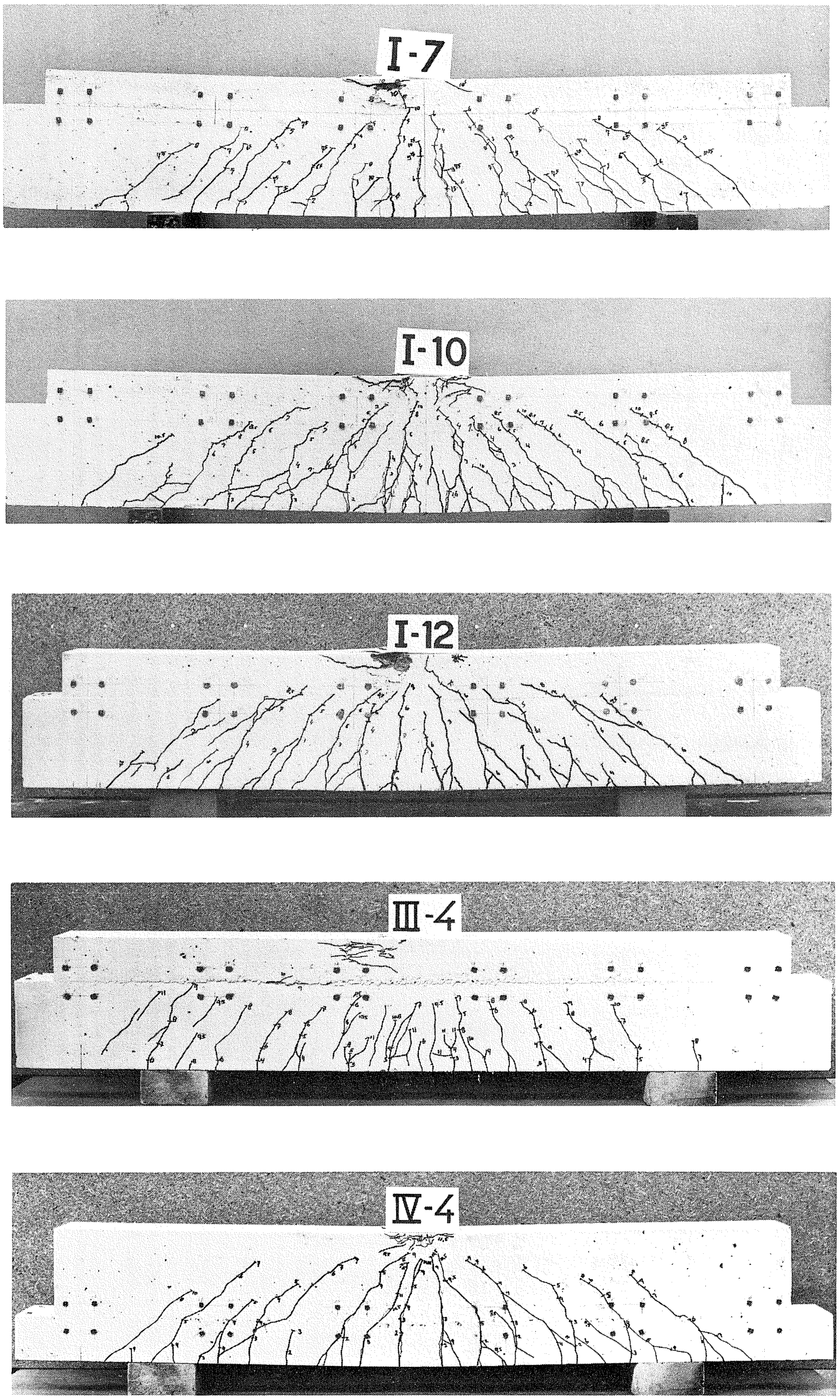


Fig. A7. Failure patterns of the beams I-7, I-10, I-12, III-4 and IV-4.

Table A1. Summary of the data and test results.

1	2	3	4	5	6	7	8	9	10	11	1
				main reinforcement			stirrup reinforcement				
beam No.	relative height of interface (h_1/h)	beam component	f'_{cm} (N/mm²)	⌀ (mm)	ω_0 (%)	$f_{0,2}$ (N/mm²)	⌀ (mm)	f_{et} (N/mm²)	$100\omega_{tv}$	$100\omega_{t1}$	1
I-1	0,72	2 1	20,7 47,7	3⌀10	1,24	447	⌀6–70	266	0,78	0,81	0
I-2	0,44	2 1	21,8 54,5	3⌀10	1,24	447	⌀6–70	266	0,91	0,81	0
I-3	0,44	2 1	21,8 54,5	3⌀10	1,24	447	⌀6–70	266	0,80	0	0,
I-4	0,16	2 1	21,8 54,5	3⌀10	1,24	447	–	–	(0,78)	0	0
I-5	0,16	2 1	21,8 54,5	3⌀10	1,24	447	⌀6–70	266	0,80	0	0,
I-6	0,72	2 1	30,5 30,2	2⌀12+ 1⌀10	1,63	500/447	⌀6–55	266	1,10	1,03	0
I-7	0,72	2 1	30,5 30,2	2⌀12+ 1⌀10	1,63	500/447	⌀6–55	266	1,18	0	1,
I-8	0,72	2 1	20,7 47,7	3⌀10	1,24	447	⌀6–70	266	0,79	0	0,
I-9	monolithic		30,5	2⌀12+ 1⌀10	1,63	500/447	⌀6–55	266	1,10	1,03	(C
I-10	monolithic		32,8	2⌀12+ 1⌀10	1,63	500/447	⌀6–55	266	1,14	0	(1
I-11	0,62	2 1	32,1 30,1	2⌀12+ 1⌀10	1,64	486/447	⌀6–55	305	1,02	0,72	0,
I-12	0,62	2 1	32,1 30,1	2⌀12+ 1⌀10	1,64	486/447	⌀6–55	305	1,02	0,30	0,
I-13	0,44	2 1	19,7 53,9	3⌀10	1,24	434	⌀6–70	303	0,78	0,51	0,
I-14	0,44	2 1	19,7 53,9	3⌀10	1,24	434	⌀6–70	303	0,78	0,29	0,
I-15	monolithic		32,0	2⌀12+ 1⌀10	1,67	470/434	⌀6–105	303	0,97	0	(C
I-16	0,62	2 1	32,0 33,0	2⌀12+ 1⌀10	1,67	470/434	⌀6–105	303	0,97	0	0,
I-17	0,62	2 1	32,7 34,3	2⌀12+ 1⌀10	1,67	470/434	⌀6–75	303	0,98	0	0,
II-1	0,56	2 1	37,4 37,9	2⌀12+ 1⌀10	1,27	486/447	⌀6–65	305	1,11	0,94	0
II-2	0,56	2 1	37,4 37,9	2⌀12+ 1⌀10	1,27	486/447	⌀6–65	305	1,11	0,72	0,
II-3	0,56	2 1	37,4 37,9	2⌀12+ 1⌀10	1,27	486/447	⌀6–65	305	1,11	0,50	0,

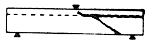
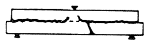
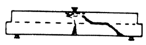
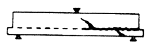
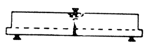
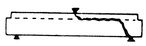
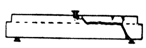
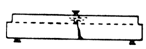
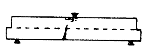
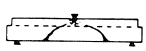
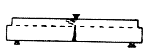
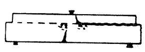
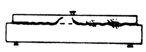
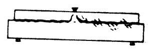
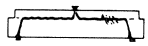
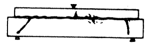
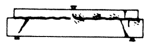
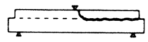
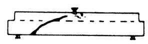
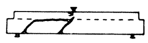
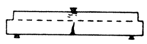
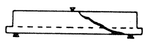
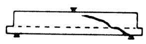
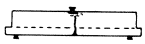
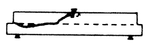
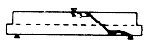
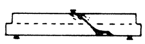
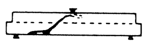
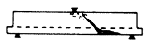
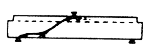
	14	15	16	17	18	19	20	21	22	23	24
oretical					measured						
ure ment (lm)	failure load F_u (kN)	shear strength T_{u1} (kN)	T_{u2} (kN)	failure load $F_{us} = 2(T_{u1}+T_{u2})$ (kN)	load at first horizontal crack F_h (kN)	failure load F_{br} (kN)	$\frac{F_{br}}{F_u}$	$\frac{F_{br}}{F_{us}}$	$\tau_u = \frac{\frac{1}{2}F_{br}}{bh}$ (N/mm²)	$\frac{\tau_u}{f_{bm}}$	failure pattern
31	61,2	24,8	0	49,6	52	65	1,06	1,31	1,82	0,90	
)4	64,2	15,3	0	30,6	30	38	0,59	1,24	1,06	0,51	
36	65,4	0	34,6	69,2	45	72,5	1,11	1,05	2,03	0,97	
41	61,6	0	0	(22,4)	30	30	0,49	1,34	0,84	0,40	
36	65,4	0	34,6	69,2	30	80	1,22	1,16	2,23	1,07	
54	86,2	31,5	0	63,0	60	60	0,67	0,95	1,69	0,67	
48	93,9	0	41,3	82,6	–	100	1,06	1,21	2,81	1,12	
3	64,5	0	34,7	69,4	77,5	77,5	1,20	1,12	2,16	1,06	
54	86,2	31,5	0	63,0	(90)	90	1,04	1,43	2,53	1,00	
54	89,4	0	41,3	82,6	–	107,5	1,20	1,30	3,02	1,14	
)3	87,7	21,9	15,1	74,0	70	90	1,03	1,22	2,53	1,01	
)3	87,7	9,1	36,6	91,4	85	115	1,31	1,26	3,23	1,29	
52	66,5	11,0	15,6	53,2	45	62,5	0,94	1,17	1,76	0,89	
52	66,5	6,25	26,4	65,3	65	70	1,05	1,07	1,97	1,00	
4	90,2	0	26,4	52,8	–	106	1,18	2,00	2,98	1,15	
4	90,2	0	26,4	52,8	90	102,5	1,14	1,94	2,88	1,11	
0	90,8	0	37,1	74,2	100	100	1,10	1,35	2,81	1,07	
9	127,6	33,3	0	66,6	(0)	66	0,52	0,99	1,45	0,51	
9	127,6	25,5	13,8	78,6	(0)	70	0,55	0,89	1,54	0,54	
9	127,6	17,7	27,5	90,4	(0)	85	0,67	0,94	1,86	0,65	

Table A1. (continued)

1	2	3	4	5	6	7	8	9	10	11	12
				main reinforcement			stirrup reinforcement				
beam No.	relative height of interface (h_1/h)	beam component	f'_{cm} (N/mm²)	⌀ (mm)	ω_0 (%)	$f_{0,2}$ (N/mm²)	⌀ (mm)	f_{et} (N/mm²)	$100\omega_{tv}$	$100\omega_{t1}$	10
II-4	0,56	2 1	37,3 38,4	2⌀12+ 1⌀10	1,27	486/447	⌀6–65	305	1,11	0,28	0,6
II-5	0,56	2 1	37,3 38,4	2⌀12+ 1⌀10	1,27	486/447	⌀6–65	305	1,11	0	0,9
II-6	0,56	2 1	32,1 30,1	2⌀12+ 1⌀10	1,27	486/447	⌀6–52,5	305	1,09	0	1,0
III-1	0,62	2 1	32,4 33,0	2⌀12+ 1⌀10	1,67	470/434	⌀6–75 at 45°	303	1,02	1,07	0
III-2	0,62	2 1	29,2 32,0	2⌀12+ 1⌀10	1,67	470/434	⌀6–75 at 45°	303	1,02	0,62	0,4
III-3	0,62	2 1	32,4 33,0	2⌀12+ 1⌀10	1,67	470/434	⌀6–75 at 45°	303	1,09	0,47	0,
III-4	0,62	2 1	29,2 32,0	2⌀12+ 1⌀10	1,67	470/434	⌀6–75 at 45°	303	1,02	0	1,
IV-1	0,25	2 1	32,6 29,7	2⌀12+ 1⌀10	1,67	470/434	–	–	0,95	0	0
IV-2	0,25	2 1	32,6 29,7	2⌀12+ 1⌀10	1,67	470/434	–	–	0,95	0	0
IV-3	0,25	2 1	32,6 28,1	2⌀12+ 1⌀10	1,67	470/434	⌀6–105	303	0,97	0	0,
IV-4	0,25	2 1	32,6 28,1	2⌀12+ 1⌀10	1,67	470/434	⌀6–55	303	0,97	0	1,
V-1	0,50	2 1	25,4 31,5	2⌀12+ 1⌀10	1,69	467/432	⌀6–70	313	0,96	0,82	0
V-2	0,50	2 1	25,4 31,5	2⌀12+ 1⌀10	1,69	467/432	⌀6–70	313	1,01	0,55	0,
V-3	0,50	2 1	26,4 32,2	2⌀12+ 1⌀10	1,69	467/432	⌀6–70	313	1,01	0,27	0,
V-4	0,50	2 1	27,2 33,0	2⌀12+ 1⌀10	1,69	467/432	⌀6–70	313	1,01	0	0,
V-5	0,16	2 1	29,7 35,4	2⌀12+ 1⌀10	1,69	467/432	⌀6–105	313	0,94	0	0,
V-6	0,72	2 1	30,5 36,0	2⌀12+ 1⌀10	1,69	467/432	⌀6–70	313	0,94	0,27	0,

Explanatory notes: Column 10. The requisite shear reinforcement has been determined from: $\omega_{tv} = 2M_u/l/bzf_{et}$

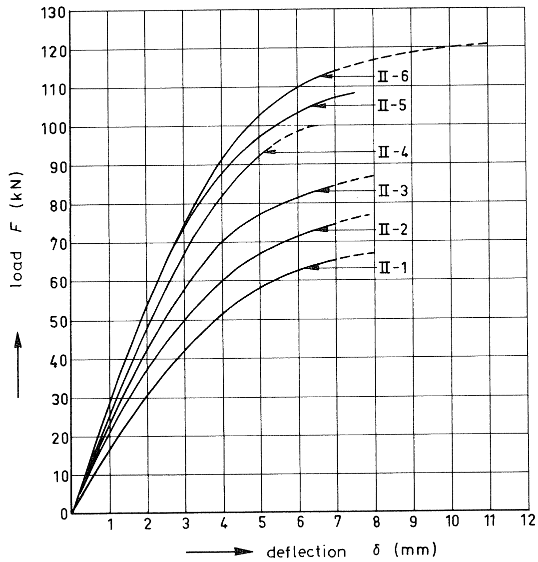
Column 13. The theoretical failure moment has been determined from the internal equilibrium at the mid-span section, adopting the actual concrete and steel strengths

	14	15	16	17	18	19	20	21	22	23	24
eoretical					measured						
ilure oment $_u$ Nm)	failure load F_u (kN)	shear strength T_{u1} (kN)	T_{u2} (kN)	failure load $F_{us} =$ $2(T_{u1}+T_{u2})$ (kN)	load at first horizontal crack F_h (kN)	failure load F_{br} (kN)	$\frac{F_{br}}{F_u}$	$\frac{F_{br}}{F_{us}}$	$\tau_u = \frac{\frac{1}{2}F_{br}}{bh}$ (N/mm²)	$\frac{\tau_u}{f_{bm}}$	failure pattern
,89	127,6	9,9	41,3	102,4	(0)	95	0,74	0,93	2,08	0,72	
,89	127,6	0	58,9	117,8	(0)	109	0,85	0,93	2,39	0,83	
,51	122,0	0	67,0	134,0	(0)	115	0,90	0,86	2,52	1,01	
,79	87,2	32,4	0	64,8	20	75	0,86	1,16	2,11	0,81	
,10	88,4	18,8	21,8	81,2	20	107,5	1,22	1,32	3,02	1,23	
,90	91,6	14,2	29,2	86,8	30	110	1,20	1,27	3,09	1,18	
,10	88,4	0	52,0	104,0	30	112,5	1,27	1,08	3,16	1,28	
,65	86,6	0	0	(28,1)	25	52	0,60	1,85	1,46	0,59	
,65	86,6	0	0	(28,1)	20	40	0,46	1,42	1,12	0,45	
,67	90,7	0	26,2	52,4	20	102,5	1,13	1,95	2,88	1,20	
67	90,7	0	51,0	102,0	20	112,5	1,24	1,10	3,16	1,32	
11	80,4	20,3	0	40,6	40	47,5	0,55	1,17	1,34	0,59	
72	86,9	13,65	13,5	54,3	40	70	0,81	1,29	1,97	0,87	
72	86,9	6,7	27,6	68,6	30	80	0,92	1,17	2,25	0,97	
72	86,9	0	41,1	82,2	50	85	0,98	1,03	2,39	1,01	
44	89,8	0	27,6	55,2	40	80	0,89	1,45	2,25	0,91	
44	89,8	9,75	27,6	74,7	75	85	0,95	1,14	2,39	0,95	

Column 14. $F_u = 4M_u/l$

Column 17. Only in the case where there are no stirrups at all the following has been adopted: $F_{us} = 2T_{us} = 2 \cdot 0{,}3 f_{bm} bh$

Column 23. The tensile strength of the concrete is obtained from: $f_{bm} = 1 + \frac{1}{20} \cdot f'_{cm}$



stirrup reinforcement		
nr.	$100\,\omega_{t1}$	$100\,\omega_{t2}$
II-6	0	1,07
II-5	0	0,94
II-4	0,28	0,66
II-3	0,50	0,44
II-2	0,72	0,22
II-1	0,94	0

Fig. A8. Deflections of the beams in series II.

is more gradual, so that it can justifiably be presumed that the difference between the curves of a composite beam and a monolithic beam, respectively, provides an indication of the magnitude of the slip at the interface.

As was to be expected, the beams of series II constitute an exception to this; the horizontal deformation in this series is virtually constant all along the beam (gauge lengths 1, 2 and 3: see Fig. A10).

A4 Comparison between theory and test results

Deflection

A comparison between the theoretical and the measured deflections of the test beams is presented in Figs. A11 and A12. For the purpose of this comparison the theoretical deflections were calculated from the theoretical stiffness of monolithic beams in, respectively, the uncracked stage. the cracked stage and at failure. For the deflection in the service stage the theoretical relationship is found to provide a sufficiently accurate approximation (Fig. A11). An entirely different set of conditions arises if it is assumed that shearing is possible along the entire length of the interface. In that case the stiffness of the composite beam will also be dependent upon the resistance that the connecting reinforcement can develop at the interface. The calculation of the deflection curves in the various stages is rather laborious, but the results is satisfactory (Fig. A12).

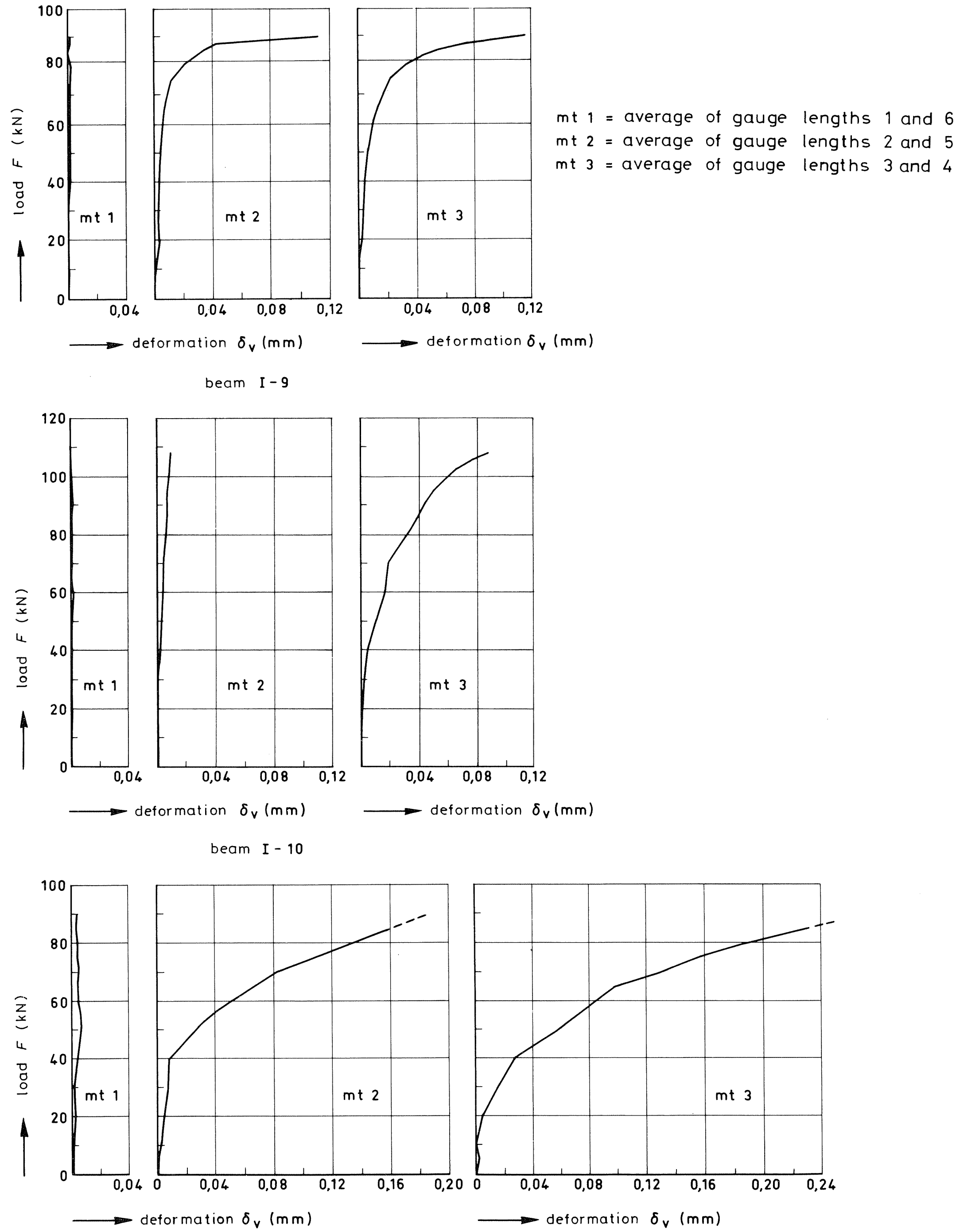


Fig. A9. Horizontal deformation of the beams I-9, I-10 and I-11.

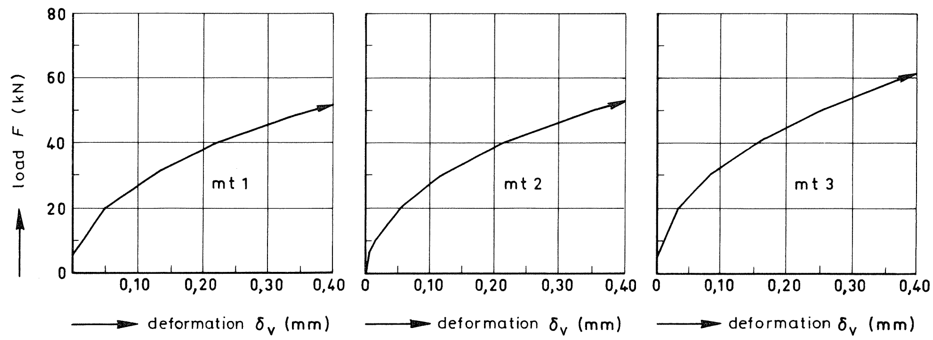


Fig. A10. Horizontal deformation of beam II-3 ($F_{br} = 85$ kN).

Strength

A preliminary indication of the strength is obtained from the ratio F_{br}/F_u in colomn 20 of Table A1. The value $F_u = 4M_u/l$ has been adopted as the theoretical failure load F_u. It appears that when 100% connecting reinforcement is provided, i.e., when $\omega_{t2} \approx \omega_{tv}$, the theoretical strength of a monolithic beam is amply attained in most cases. Basing oneself on the double truss analogy (Fig. 3), it has been assumed as a working hypothesis that the strength (load capacity) of a composite beam is equal to the sum of the load capacities of the two separate trusses.

Next, the values T_{u1} and T_{u2} were calculated. They are listed in colomns 15 and 16 of Table A1. In Fig. A13 the values of F_{br}/F_u have been plotted against F_{us}/F_u for all the beams. Although there is fairly wide scatter, it can permissibly be inferred that formula (4) constitutes a good lower bound for the theoretical failure load of a composite beam.

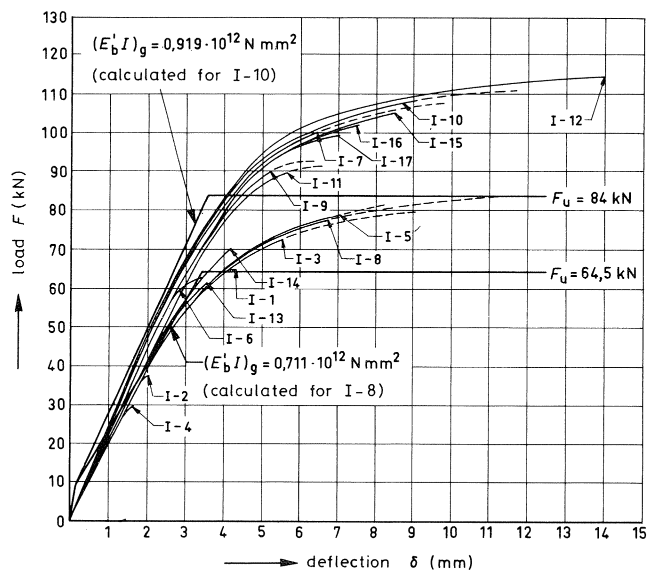


Fig. A11. Comparison of theoretical and measured deflections of the beams in series I.

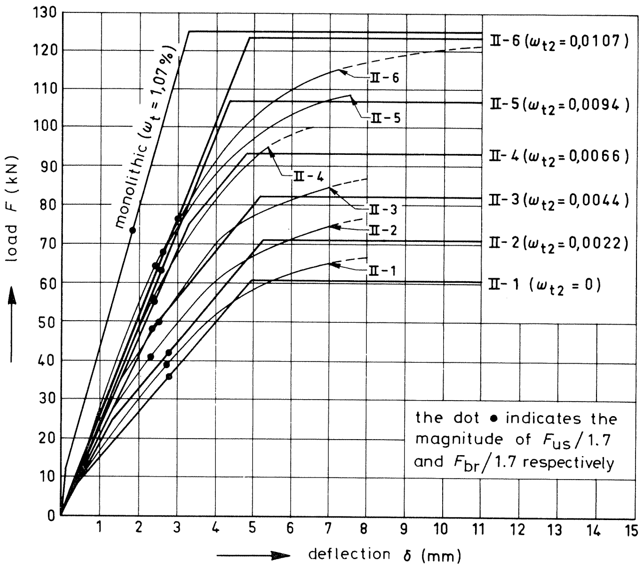
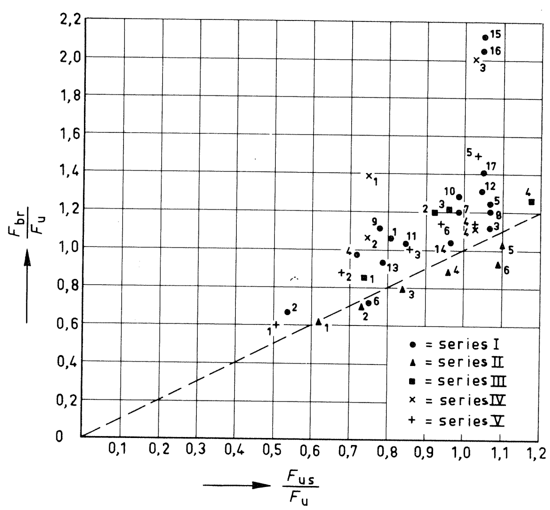


Fig. A12. Comparison of theoretical and measured deflections of the beams in series II.



F_{us} = theoretische bezwijklast volgens $F_{us} = 2(F_{u1} + F_{u2})$

F_{br} = gemeten bezwijklast

F_u = theoretische bezwijklast van een identieke monoliet

Fig. A13. Relations between F_{us}/F_u and F_{br}/F_u.